

# Mathematica 11.3 Integration Test Results

Test results for the 1917 problems in "1.1.1.2 (a+b x)^m (c+d x)^n.m"

Problem 73: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^3}{x^5} dx$$

Optimal (type 1, 17 leaves, 1 step):

$$-\frac{(a + b x)^4}{4 a x^4}$$

Result (type 1, 39 leaves):

$$-\frac{a^3}{4 x^4} - \frac{a^2 b}{x^3} - \frac{3 a b^2}{2 x^2} - \frac{b^3}{x}$$

Problem 82: Result more than twice size of optimal antiderivative.

$$\int x (a + b x)^5 dx$$

Optimal (type 1, 30 leaves, 2 steps):

$$-\frac{a (a + b x)^6}{6 b^2} + \frac{(a + b x)^7}{7 b^2}$$

Result (type 1, 67 leaves):

$$\frac{a^5 x^2}{2} + \frac{5}{3} a^4 b x^3 + \frac{5}{2} a^3 b^2 x^4 + 2 a^2 b^3 x^5 + \frac{5}{6} a b^4 x^6 + \frac{b^5 x^7}{7}$$

Problem 90: Result more than twice size of optimal antiderivative.

$$\int \frac{(a + b x)^5}{x^7} dx$$

Optimal (type 1, 17 leaves, 1 step):

$$-\frac{(a + b x)^6}{6 a x^6}$$

Result (type 1, 65 leaves):

$$-\frac{a^5}{6 x^6} - \frac{a^4 b}{x^5} - \frac{5 a^3 b^2}{2 x^4} - \frac{10 a^2 b^3}{3 x^3} - \frac{5 a b^4}{2 x^2} - \frac{b^5}{x}$$

**Problem 105: Result more than twice size of optimal antiderivative.**

$$\int x (a + b x)^7 dx$$

Optimal (type 1, 30 leaves, 2 steps):

$$-\frac{a (a + b x)^8}{8 b^2} + \frac{(a + b x)^9}{9 b^2}$$

Result (type 1, 91 leaves):

$$\frac{a^7 x^2}{2} + \frac{7}{3} a^6 b x^3 + \frac{21}{4} a^5 b^2 x^4 + 7 a^4 b^3 x^5 + \frac{35}{6} a^3 b^4 x^6 + 3 a^2 b^5 x^7 + \frac{7}{8} a b^6 x^8 + \frac{b^7 x^9}{9}$$

**Problem 115: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^7}{x^9} dx$$

Optimal (type 1, 17 leaves, 1 step):

$$-\frac{(a + b x)^8}{8 a x^8}$$

Result (type 1, 87 leaves):

$$-\frac{a^7}{8 x^8} - \frac{a^6 b}{x^7} - \frac{7 a^5 b^2}{2 x^6} - \frac{7 a^4 b^3}{x^5} - \frac{35 a^3 b^4}{4 x^4} - \frac{7 a^2 b^5}{x^3} - \frac{7 a b^6}{2 x^2} - \frac{b^7}{x}$$

**Problem 116: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^7}{x^{10}} dx$$

Optimal (type 1, 36 leaves, 2 steps):

$$-\frac{(a + b x)^8}{9 a x^9} + \frac{b (a + b x)^8}{72 a^2 x^8}$$

Result (type 1, 91 leaves):

$$-\frac{a^7}{9 x^9} - \frac{7 a^6 b}{8 x^8} - \frac{3 a^5 b^2}{x^7} - \frac{35 a^4 b^3}{6 x^6} - \frac{7 a^3 b^4}{x^5} - \frac{21 a^2 b^5}{4 x^4} - \frac{7 a b^6}{3 x^3} - \frac{b^7}{2 x^2}$$

**Problem 132: Result more than twice size of optimal antiderivative.**

$$\int x^2 (a + b x)^{10} dx$$

Optimal (type 1, 47 leaves, 2 steps):

$$\frac{a^2 (a + b x)^{11}}{11 b^3} - \frac{a (a + b x)^{12}}{6 b^3} + \frac{(a + b x)^{13}}{13 b^3}$$

Result (type 1, 126 leaves):

$$\frac{a^{10} x^3}{3} + \frac{5}{2} a^9 b x^4 + 9 a^8 b^2 x^5 + 20 a^7 b^3 x^6 + 30 a^6 b^4 x^7 + \frac{63}{2} a^5 b^5 x^8 + \frac{70}{3} a^4 b^6 x^9 + 12 a^3 b^7 x^{10} + \frac{45}{11} a^2 b^8 x^{11} + \frac{5}{6} a b^9 x^{12} + \frac{b^{10} x^{13}}{13}$$

**Problem 133: Result more than twice size of optimal antiderivative.**

$$\int x (a + b x)^{10} dx$$

Optimal (type 1, 30 leaves, 2 steps):

$$-\frac{a (a + b x)^{11}}{11 b^2} + \frac{(a + b x)^{12}}{12 b^2}$$

Result (type 1, 128 leaves):

$$\frac{a^{10} x^2}{2} + \frac{10}{3} a^9 b x^3 + \frac{45}{4} a^8 b^2 x^4 + 24 a^7 b^3 x^5 + 35 a^6 b^4 x^6 + 36 a^5 b^5 x^7 + \frac{105}{4} a^4 b^6 x^8 + \frac{40}{3} a^3 b^7 x^9 + \frac{9}{2} a^2 b^8 x^{10} + \frac{10}{11} a b^9 x^{11} + \frac{b^{10} x^{12}}{12}$$

**Problem 146: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{10}}{x^{12}} dx$$

Optimal (type 1, 17 leaves, 1 step):

$$-\frac{(a + b x)^{11}}{11 a x^{11}}$$

Result (type 1, 114 leaves):

$$-\frac{a^{10}}{11 x^{11}} - \frac{a^9 b}{x^{10}} - \frac{5 a^8 b^2}{x^9} - \frac{15 a^7 b^3}{x^8} - \frac{30 a^6 b^4}{x^7} - \frac{42 a^5 b^5}{x^6} - \frac{42 a^4 b^6}{x^5} - \frac{30 a^3 b^7}{x^4} - \frac{15 a^2 b^8}{x^3} - \frac{5 a b^9}{x^2} - \frac{b^{10}}{x}$$

**Problem 147: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{10}}{x^{13}} dx$$

Optimal (type 1, 36 leaves, 2 steps):

$$-\frac{(a + b x)^{11}}{12 a x^{12}} + \frac{b (a + b x)^{11}}{132 a^2 x^{11}}$$

Result (type 1, 128 leaves):

$$\frac{a^{10}}{12 x^{12}} - \frac{10 a^9 b}{11 x^{11}} - \frac{9 a^8 b^2}{2 x^{10}} - \frac{40 a^7 b^3}{3 x^9} - \frac{105 a^6 b^4}{4 x^8} - \frac{36 a^5 b^5}{x^7} - \frac{35 a^4 b^6}{x^6} - \frac{24 a^3 b^7}{x^5} - \frac{45 a^2 b^8}{4 x^4} - \frac{10 a b^9}{3 x^3} - \frac{b^{10}}{2 x^2}$$

**Problem 148: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^{10}}{x^{14}} dx$$

Optimal (type 1, 56 leaves, 3 steps):

$$-\frac{(a + b x)^{11}}{13 a x^{13}} + \frac{b (a + b x)^{11}}{78 a^2 x^{12}} - \frac{b^2 (a + b x)^{11}}{858 a^3 x^{11}}$$

Result (type 1, 126 leaves):

$$-\frac{a^{10}}{13 x^{13}} - \frac{5 a^9 b}{6 x^{12}} - \frac{45 a^8 b^2}{11 x^{11}} - \frac{12 a^7 b^3}{x^{10}} - \frac{70 a^6 b^4}{3 x^9} - \frac{63 a^5 b^5}{2 x^8} - \frac{30 a^4 b^6}{x^7} - \frac{20 a^3 b^7}{x^6} - \frac{9 a^2 b^8}{x^5} - \frac{5 a b^9}{2 x^4} - \frac{b^{10}}{3 x^3}$$

**Problem 212: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^5}{(a + b x)^7} dx$$

Optimal (type 1, 17 leaves, 1 step):

$$\frac{x^6}{6 a (a + b x)^6}$$

Result (type 1, 64 leaves):

$$-\frac{a^5 + 6 a^4 b x + 15 a^3 b^2 x^2 + 20 a^2 b^3 x^3 + 15 a b^4 x^4 + 6 b^5 x^5}{6 b^6 (a + b x)^6}$$

**Problem 226: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^8}{(a + b x)^{10}} dx$$

Optimal (type 1, 17 leaves, 1 step):

$$\frac{x^9}{9 a (a + b x)^9}$$

Result (type 1, 97 leaves):

$$-\frac{1}{9b^9(a+bx)^9} (a^8 + 9a^7bx + 36a^6b^2x^2 + 84a^5b^3x^3 + 126a^4b^4x^4 + 126a^3b^5x^5 + 84a^2b^6x^6 + 36ab^7x^7 + 9b^8x^8)$$

**Problem 227: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^7}{(a+bx)^{10}} dx$$

Optimal (type 1, 35 leaves, 2 steps):

$$\frac{x^8}{9a(a+bx)^9} + \frac{x^8}{72a^2(a+bx)^8}$$

Result (type 1, 86 leaves):

$$-\frac{1}{72b^8(a+bx)^9} (a^7 + 9a^6bx + 36a^5b^2x^2 + 84a^4b^3x^3 + 126a^3b^4x^4 + 126a^2b^5x^5 + 84ab^6x^6 + 36b^7x^7)$$

**Problem 243: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^8}{x^{10}} dx$$

Optimal (type 1, 17 leaves, 1 step):

$$-\frac{(a+bx)^9}{9ax^9}$$

Result (type 1, 96 leaves):

$$\frac{a^8}{9x^9} - \frac{a^7b}{x^8} - \frac{4a^6b^2}{x^7} - \frac{28a^5b^3}{3x^6} - \frac{14a^4b^4}{x^5} - \frac{14a^3b^5}{x^4} - \frac{28a^2b^6}{3x^3} - \frac{4ab^7}{x^2} - \frac{b^8}{x}$$

**Problem 244: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a+bx)^7}{x^{10}} dx$$

Optimal (type 1, 36 leaves, 2 steps):

$$-\frac{(a+bx)^8}{9ax^9} + \frac{b(a+bx)^8}{72a^2x^8}$$

Result (type 1, 91 leaves):

$$\frac{a^7}{9x^9} - \frac{7a^6b}{8x^8} - \frac{3a^5b^2}{x^7} - \frac{35a^4b^3}{6x^6} - \frac{7a^3b^4}{x^5} - \frac{21a^2b^5}{4x^4} - \frac{7ab^6}{3x^3} - \frac{b^7}{2x^2}$$

**Problem 368: Result unnecessarily involves higher level functions and more**

than twice size of optimal antiderivative.

$$\int \frac{x^{-1+m} (2 a m + b (-1 + 2 m) x)}{2 (a + b x)^{3/2}} dx$$

Optimal (type 3, 13 leaves, 2 steps):

$$\frac{x^m}{\sqrt{a + b x}}$$

Result (type 5, 100 leaves):

$$\left( x^m \sqrt{a + b x} \left( 2 a (1 + m) \text{Hypergeometric2F1} \left[ -\frac{1}{2}, m, 1 + m, -\frac{b x}{a} \right] - \right. \right. \\ \left. \left. b x \left( 2 m \text{Hypergeometric2F1} \left[ \frac{1}{2}, 1 + m, 2 + m, -\frac{b x}{a} \right] + \right. \right. \right. \\ \left. \left. \left. \text{Hypergeometric2F1} \left[ \frac{3}{2}, 1 + m, 2 + m, -\frac{b x}{a} \right] \right) \right) \right) / \left( 2 a^2 (1 + m) \sqrt{1 + \frac{b x}{a}} \right)$$

**Problem 369:** Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \left( -\frac{b x^m}{2 (a + b x)^{3/2}} + \frac{m x^{-1+m}}{\sqrt{a + b x}} \right) dx$$

Optimal (type 3, 13 leaves, ? steps):

$$\frac{x^m}{\sqrt{a + b x}}$$

Result (type 5, 100 leaves):

$$\left( x^m \sqrt{a + b x} \left( 2 a (1 + m) \text{Hypergeometric2F1} \left[ -\frac{1}{2}, m, 1 + m, -\frac{b x}{a} \right] - \right. \right. \\ \left. \left. b x \left( 2 m \text{Hypergeometric2F1} \left[ \frac{1}{2}, 1 + m, 2 + m, -\frac{b x}{a} \right] + \right. \right. \right. \\ \left. \left. \left. \text{Hypergeometric2F1} \left[ \frac{3}{2}, 1 + m, 2 + m, -\frac{b x}{a} \right] \right) \right) \right) / \left( 2 a^2 (1 + m) \sqrt{1 + \frac{b x}{a}} \right)$$

**Problem 375:** Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1/3}}{x} dx$$

Optimal (type 3, 91 leaves, 5 steps):

$$3 (a + b x)^{1/3} - \sqrt{3} a^{1/3} \operatorname{ArcTan}\left[\frac{a^{1/3} + 2 (a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right] - \frac{1}{2} a^{1/3} \operatorname{Log}[x] + \frac{3}{2} a^{1/3} \operatorname{Log}[a^{1/3} - (a + b x)^{1/3}]$$

Result (type 5, 57 leaves):

$$\frac{6 (a + b x) - 3 a \left(1 + \frac{a}{b x}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x}\right]}{2 (a + b x)^{2/3}}$$

**Problem 376: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{1/3}}{x^2} dx$$

Optimal (type 3, 97 leaves, 5 steps):

$$-\frac{(a + b x)^{1/3}}{x} - \frac{b \operatorname{ArcTan}\left[\frac{a^{1/3} + 2 (a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{2/3}} - \frac{b \operatorname{Log}[x]}{6 a^{2/3}} + \frac{b \operatorname{Log}[a^{1/3} - (a + b x)^{1/3}]}{2 a^{2/3}}$$

Result (type 5, 61 leaves):

$$\frac{-2 (a + b x) - b \left(1 + \frac{a}{b x}\right)^{2/3} x \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x}\right]}{2 x (a + b x)^{2/3}}$$

**Problem 377: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{1/3}}{x^3} dx$$

Optimal (type 3, 127 leaves, 6 steps):

$$-\frac{(a + b x)^{1/3}}{2 x^2} - \frac{b (a + b x)^{1/3}}{6 a x} + \frac{b^2 \operatorname{ArcTan}\left[\frac{a^{1/3} + 2 (a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{5/3}} + \frac{b^2 \operatorname{Log}[x]}{18 a^{5/3}} - \frac{b^2 \operatorname{Log}[a^{1/3} - (a + b x)^{1/3}]}{6 a^{5/3}}$$

Result (type 5, 78 leaves):

$$\frac{1}{6 a x^2 (a + b x)^{2/3}} \left( -3 a^2 - 4 a b x - b^2 x^2 + b^2 \left(1 + \frac{a}{b x}\right)^{2/3} x^2 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x}\right] \right)$$

**Problem 382: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{2/3}}{x} dx$$

Optimal (type 3, 92 leaves, 5 steps):

$$\frac{3}{2} (a + b x)^{2/3} + \sqrt{3} a^{2/3} \operatorname{ArcTan}\left[\frac{a^{1/3} + 2 (a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right] - \frac{1}{2} a^{2/3} \operatorname{Log}[x] + \frac{3}{2} a^{2/3} \operatorname{Log}[a^{1/3} - (a + b x)^{1/3}]$$

Result (type 5, 57 leaves):

$$\frac{3(a+bx) - 6a\left(1 + \frac{a}{bx}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{bx}\right]}{2(a+bx)^{1/3}}$$

**Problem 383: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{2/3}}{x^2} dx$$

Optimal (type 3, 94 leaves, 5 steps):

$$-\frac{(a+bx)^{2/3}}{x} + \frac{2b \text{ArcTan}\left[\frac{a^{1/3}+2(a+bx)^{1/3}}{\sqrt{3}a^{1/3}}\right]}{\sqrt{3}a^{1/3}} - \frac{b \text{Log}[x]}{3a^{1/3}} + \frac{b \text{Log}[a^{1/3} - (a+bx)^{1/3}]}{a^{1/3}}$$

Result (type 5, 58 leaves):

$$\frac{-a - bx - 2b\left(1 + \frac{a}{bx}\right)^{1/3} x \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{bx}\right]}{x(a+bx)^{1/3}}$$

**Problem 384: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{2/3}}{x^3} dx$$

Optimal (type 3, 127 leaves, 6 steps):

$$-\frac{(a+bx)^{2/3}}{2x^2} - \frac{b(a+bx)^{2/3}}{3ax} - \frac{b^2 \text{ArcTan}\left[\frac{a^{1/3}+2(a+bx)^{1/3}}{\sqrt{3}a^{1/3}}\right]}{3\sqrt{3}a^{4/3}} + \frac{b^2 \text{Log}[x]}{18a^{4/3}} - \frac{b^2 \text{Log}[a^{1/3} - (a+bx)^{1/3}]}{6a^{4/3}}$$

Result (type 5, 79 leaves):

$$\frac{1}{6ax^2(a+bx)^{1/3}} \left( -3a^2 - 5abx - 2b^2x^2 + 2b^2\left(1 + \frac{a}{bx}\right)^{1/3} x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{bx}\right] \right)$$

**Problem 389: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{4/3}}{x} dx$$

Optimal (type 3, 105 leaves, 6 steps):

$$3a(a+bx)^{1/3} + \frac{3}{4}(a+bx)^{4/3} - \sqrt{3}a^{4/3} \text{ArcTan}\left[\frac{a^{1/3}+2(a+bx)^{1/3}}{\sqrt{3}a^{1/3}}\right] - \frac{1}{2}a^{4/3} \text{Log}[x] + \frac{3}{2}a^{4/3} \text{Log}[a^{1/3} - (a+bx)^{1/3}]$$

Result (type 5, 74 leaves):



$$\left(\frac{15a}{4} + \frac{3bx}{4}\right) (a+bx)^{1/3} - \frac{3a^2 \left(\frac{a+bx}{bx}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{bx}\right]}{2(a+bx)^{2/3}}$$

**Problem 390: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{4/3}}{x^2} dx$$

Optimal (type 3, 107 leaves, 6 steps):

$$4b(a+bx)^{1/3} - \frac{(a+bx)^{4/3}}{x} - \frac{4a^{1/3}b \text{ArcTan}\left[\frac{a^{1/3}+2(a+bx)^{1/3}}{\sqrt{3}a^{1/3}}\right]}{\sqrt{3}} - \frac{2}{3}a^{1/3}b \text{Log}[x] + 2a^{1/3}b \text{Log}\left[a^{1/3} - (a+bx)^{1/3}\right]$$

Result (type 5, 64 leaves):

$$\frac{1}{(a+bx)^{2/3}} \left( \left(3b - \frac{a}{x}\right) (a+bx) - 2ab \left(1 + \frac{a}{bx}\right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{bx}\right] \right)$$

**Problem 391: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{4/3}}{x^3} dx$$

Optimal (type 3, 124 leaves, 6 steps):

$$-\frac{2b(a+bx)^{1/3}}{3x} - \frac{(a+bx)^{4/3}}{2x^2} - \frac{2b^2 \text{ArcTan}\left[\frac{a^{1/3}+2(a+bx)^{1/3}}{\sqrt{3}a^{1/3}}\right]}{3\sqrt{3}a^{2/3}} - \frac{b^2 \text{Log}[x]}{9a^{2/3}} + \frac{b^2 \text{Log}\left[a^{1/3} - (a+bx)^{1/3}\right]}{3a^{2/3}}$$

Result (type 5, 76 leaves):

$$\frac{1}{6x^2(a+bx)^{2/3}} \left( -3a^2 - 10abx - 7b^2x^2 - 2b^2 \left(1 + \frac{a}{bx}\right)^{2/3} x^2 \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{bx}\right] \right)$$

**Problem 396: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x(a+bx)^{1/3}} dx$$

Optimal (type 3, 79 leaves, 4 steps):

$$\frac{\sqrt{3} \text{ArcTan}\left[\frac{a^{1/3}+2(a+bx)^{1/3}}{\sqrt{3}a^{1/3}}\right]}{a^{1/3}} - \frac{\text{Log}[x]}{2a^{1/3}} + \frac{3 \text{Log}\left[a^{1/3} - (a+bx)^{1/3}\right]}{2a^{1/3}}$$

Result (type 5, 46 leaves):

$$-\frac{3 \left(\frac{a+bx}{bx}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{bx}\right]}{(a+bx)^{1/3}}$$

**Problem 397: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^2 (a+bx)^{1/3}} dx$$

Optimal (type 3, 100 leaves, 5 steps):

$$-\frac{(a+bx)^{2/3}}{ax} - \frac{b \text{ArcTan}\left[\frac{a^{1/3}+2(a+bx)^{1/3}}{\sqrt{3}a^{1/3}}\right]}{\sqrt{3}a^{4/3}} + \frac{b \text{Log}[x]}{6a^{4/3}} - \frac{b \text{Log}\left[a^{1/3} - (a+bx)^{1/3}\right]}{2a^{4/3}}$$

Result (type 5, 60 leaves):

$$\frac{-a-bx+b\left(1+\frac{a}{bx}\right)^{1/3} x \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{bx}\right]}{ax(a+bx)^{1/3}}$$

**Problem 398: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^3 (a+bx)^{1/3}} dx$$

Optimal (type 3, 130 leaves, 6 steps):

$$-\frac{(a+bx)^{2/3}}{2ax^2} + \frac{2b(a+bx)^{2/3}}{3a^2x} + \frac{2b^2 \text{ArcTan}\left[\frac{a^{1/3}+2(a+bx)^{1/3}}{\sqrt{3}a^{1/3}}\right]}{3\sqrt{3}a^{7/3}} - \frac{b^2 \text{Log}[x]}{9a^{7/3}} + \frac{b^2 \text{Log}\left[a^{1/3} - (a+bx)^{1/3}\right]}{3a^{7/3}}$$

Result (type 5, 78 leaves):

$$\frac{\left(-3a^2+abx+4b^2x^2-4b^2\left(1+\frac{a}{bx}\right)^{1/3}x^2 \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{bx}\right]\right)}{\left(6a^2x^2(a+bx)^{1/3}\right)}$$

**Problem 404: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^2 (-a+bx)^{1/3}} dx$$

Optimal (type 3, 103 leaves, 5 steps):

$$\frac{(-a+bx)^{2/3}}{ax} - \frac{b \text{ArcTan}\left[\frac{a^{1/3}-2(-a+bx)^{1/3}}{\sqrt{3}a^{1/3}}\right]}{\sqrt{3}a^{4/3}} + \frac{b \text{Log}[x]}{6a^{4/3}} - \frac{b \text{Log}\left[a^{1/3} + (-a+bx)^{1/3}\right]}{2a^{4/3}}$$

Result (type 5, 62 leaves):

$$\frac{-a + bx - b \left(1 - \frac{a}{bx}\right)^{1/3} x \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{a}{bx}\right]}{a x (-a + bx)^{1/3}}$$

**Problem 405: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^3 (-a + bx)^{1/3}} dx$$

Optimal (type 3, 136 leaves, 6 steps):

$$\frac{(-a + bx)^{2/3}}{2 a x^2} + \frac{2 b (-a + bx)^{2/3}}{3 a^2 x} - \frac{2 b^2 \operatorname{ArcTan}\left[\frac{a^{1/3} - 2(-a + bx)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{7/3}} + \frac{b^2 \operatorname{Log}[x]}{9 a^{7/3}} - \frac{b^2 \operatorname{Log}[a^{1/3} + (-a + bx)^{1/3}]}{3 a^{7/3}}$$

Result (type 5, 81 leaves):

$$\frac{\left(-3 a^2 - a b x + 4 b^2 x^2 - 4 b^2 \left(1 - \frac{a}{bx}\right)^{1/3} x^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{a}{bx}\right]\right)}{\left(6 a^2 x^2 (-a + bx)^{1/3}\right)}$$

**Problem 410: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x (a + bx)^{2/3}} dx$$

Optimal (type 3, 80 leaves, 4 steps):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{a^{1/3} + 2(a + bx)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{a^{2/3}} - \frac{\operatorname{Log}[x]}{2 a^{2/3}} + \frac{3 \operatorname{Log}[a^{1/3} - (a + bx)^{1/3}]}{2 a^{2/3}}$$

Result (type 5, 48 leaves):

$$-\frac{3 \left(\frac{a + bx}{bx}\right)^{2/3} \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{bx}\right]}{2 (a + bx)^{2/3}}$$

**Problem 411: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^2 (a + bx)^{2/3}} dx$$

Optimal (type 3, 98 leaves, 5 steps):

$$-\frac{(a + bx)^{1/3}}{a x} + \frac{2 b \operatorname{ArcTan}\left[\frac{a^{1/3} + 2(a + bx)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{\sqrt{3} a^{5/3}} + \frac{b \operatorname{Log}[x]}{3 a^{5/3}} - \frac{b \operatorname{Log}[a^{1/3} - (a + bx)^{1/3}]}{a^{5/3}}$$

Result (type 5, 60 leaves):

$$\frac{-a - b x + b \left(1 + \frac{a}{b x}\right)^{2/3} x \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x}\right]}{a x (a + b x)^{2/3}}$$

**Problem 412: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^3 (a + b x)^{2/3}} dx$$

Optimal (type 3, 130 leaves, 6 steps):

$$-\frac{(a + b x)^{1/3}}{2 a x^2} + \frac{5 b (a + b x)^{1/3}}{6 a^2 x} - \frac{5 b^2 \operatorname{ArcTan}\left[\frac{a^{1/3} + 2(a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{8/3}} - \frac{5 b^2 \operatorname{Log}[x]}{18 a^{8/3}} + \frac{5 b^2 \operatorname{Log}[a^{1/3} - (a + b x)^{1/3}]}{6 a^{8/3}}$$

Result (type 5, 79 leaves):

$$\frac{\left(-3 a^2 + 2 a b x + 5 b^2 x^2 - 5 b^2 \left(1 + \frac{a}{b x}\right)^{2/3} x^2 \operatorname{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, -\frac{a}{b x}\right]\right)}{\left(6 a^2 x^2 (a + b x)^{2/3}\right)}$$

**Problem 417: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x (a + b x)^{4/3}} dx$$

Optimal (type 3, 93 leaves, 5 steps):

$$\frac{3}{a (a + b x)^{1/3}} + \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{a^{1/3} + 2(a + b x)^{1/3}}{\sqrt{3} a^{1/3}}\right]}{a^{4/3}} - \frac{\operatorname{Log}[x]}{2 a^{4/3}} + \frac{3 \operatorname{Log}[a^{1/3} - (a + b x)^{1/3}]}{2 a^{4/3}}$$

Result (type 5, 50 leaves):

$$\frac{3 - 3 \left(1 + \frac{a}{b x}\right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{b x}\right]}{a (a + b x)^{1/3}}$$

**Problem 418: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^2 (a + b x)^{4/3}} dx$$

Optimal (type 3, 113 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{4b}{a^2(a+bx)^{1/3}} - \frac{1}{ax(a+bx)^{1/3}} - \\
 & \frac{4b \operatorname{ArcTan}\left[\frac{a^{1/3+2(a+bx)^{1/3}}}{\sqrt{3}a^{1/3}}\right]}{\sqrt{3}a^{7/3}} + \frac{2b \operatorname{Log}[x]}{3a^{7/3}} - \frac{2b \operatorname{Log}\left[a^{1/3} - (a+bx)^{1/3}\right]}{a^{7/3}}
 \end{aligned}$$

Result (type 5, 61 leaves):

$$\frac{-a - 4bx + 4b\left(1 + \frac{a}{bx}\right)^{1/3} x \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{bx}\right]}{a^2 x (a+bx)^{1/3}}$$

**Problem 419: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{x^3 (a+bx)^{4/3}} dx$$

Optimal (type 3, 149 leaves, 7 steps):

$$\begin{aligned}
 & \frac{14b^2}{3a^3(a+bx)^{1/3}} - \frac{1}{2ax^2(a+bx)^{1/3}} + \frac{7b}{6a^2x(a+bx)^{1/3}} + \\
 & \frac{14b^2 \operatorname{ArcTan}\left[\frac{a^{1/3+2(a+bx)^{1/3}}}{\sqrt{3}a^{1/3}}\right]}{3\sqrt{3}a^{10/3}} - \frac{7b^2 \operatorname{Log}[x]}{9a^{10/3}} + \frac{7b^2 \operatorname{Log}\left[a^{1/3} - (a+bx)^{1/3}\right]}{3a^{10/3}}
 \end{aligned}$$

Result (type 5, 79 leaves):

$$\frac{\left(-3a^2 + 7abx + 28b^2x^2 - 28b^2\left(1 + \frac{a}{bx}\right)^{1/3}x^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, -\frac{a}{bx}\right]\right)}{\left(6a^3x^2(a+bx)^{1/3}\right)}$$

**Problem 648: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{1-x}\sqrt{x}} dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$-\operatorname{ArcSin}[1-2x]$$

Result (type 3, 38 leaves):

$$\frac{2\sqrt{-1+x}\sqrt{x} \operatorname{Log}\left[\sqrt{-1+x} + \sqrt{x}\right]}{\sqrt{-(-1+x)x}}$$

**Problem 707: Result more than twice size of optimal antiderivative.**

$$\int x^m (a+bx)^{5/2} dx$$

Optimal (type 5, 48 leaves, 2 steps):

$$\frac{2 x^m \left(-\frac{bx}{a}\right)^{-m} (a+bx)^{7/2} \text{Hypergeometric2F1}\left[\frac{7}{2}, -m, \frac{9}{2}, 1+\frac{bx}{a}\right]}{7 b}$$

Result (type 5, 125 leaves):

$$\left(x^{1+m} \sqrt{a+bx} \left(a^2 (6+5m+m^2) \text{Hypergeometric2F1}\left[-\frac{1}{2}, 1+m, 2+m, -\frac{bx}{a}\right] + b(1+m) x \left(2a(3+m) \text{Hypergeometric2F1}\left[-\frac{1}{2}, 2+m, 3+m, -\frac{bx}{a}\right] + b(2+m) x \text{Hypergeometric2F1}\left[-\frac{1}{2}, 3+m, 4+m, -\frac{bx}{a}\right]\right)\right)\right) / \left((1+m)(2+m)(3+m) \sqrt{1+\frac{bx}{a}}\right)$$

**Problem 713: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^{2+m}}{\sqrt{a+bx}} dx$$

Optimal (type 5, 51 leaves, 2 steps):

$$\frac{2 a^2 x^m \left(-\frac{bx}{a}\right)^{-m} \sqrt{a+bx} \text{Hypergeometric2F1}\left[\frac{1}{2}, -2-m, \frac{3}{2}, 1+\frac{bx}{a}\right]}{b^3}$$

Result (type 5, 109 leaves):

$$\left(x^{1+m} \sqrt{a+bx} \left(-a(2+m) \text{Hypergeometric2F1}\left[-\frac{1}{2}, 1+m, 2+m, -\frac{bx}{a}\right] + b(1+m) x \text{Hypergeometric2F1}\left[-\frac{1}{2}, 2+m, 3+m, -\frac{bx}{a}\right] + a(2+m) \text{Hypergeometric2F1}\left[\frac{1}{2}, 1+m, 2+m, -\frac{bx}{a}\right]\right)\right) / \left(b^2 (1+m)(2+m) \sqrt{1+\frac{bx}{a}}\right)$$

**Problem 717: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^{-2+m}}{\sqrt{a+bx}} dx$$

Optimal (type 5, 49 leaves, 2 steps):

$$\frac{2 b x^m \left(-\frac{bx}{a}\right)^{-m} \sqrt{a+bx} \text{Hypergeometric2F1}\left[\frac{1}{2}, 2-m, \frac{3}{2}, 1+\frac{bx}{a}\right]}{a^2}$$

Result (type 5, 114 leaves):

$$\left( x^{-1+m} \sqrt{a+bx} \left( a^2 m (1+m) \operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, -1+m, m, -\frac{bx}{a} \right] - \right. \right. \\ \left. \left. b (-1+m) x \left( a (1+m) \operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, m, 1+m, -\frac{bx}{a} \right] - \right. \right. \right. \\ \left. \left. \left. b m x \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, 1+m, 2+m, -\frac{bx}{a} \right] \right) \right) \right) / \left( a^3 m (-1+m^2) \sqrt{1+\frac{bx}{a}} \right)$$

**Problem 718: Result more than twice size of optimal antiderivative.**

$$\int \frac{x^{-3+m}}{\sqrt{a+bx}} dx$$

Optimal (type 5, 51 leaves, 2 steps):

$$\frac{2 b^2 x^m \left( -\frac{bx}{a} \right)^{-m} \sqrt{a+bx} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, 3-m, \frac{3}{2}, 1+\frac{bx}{a} \right]}{a^3}$$

Result (type 5, 156 leaves):

$$\left( x^{-2+m} \sqrt{1+\frac{bx}{a}} \left( a^3 m (-1+m^2) \operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, -2+m, -1+m, -\frac{bx}{a} \right] - \right. \right. \\ \left. \left. b (-2+m) x \left( a^2 m (1+m) \operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, -1+m, m, -\frac{bx}{a} \right] + b (-1+m) x \right. \right. \right. \\ \left. \left. \left. \left( -a (1+m) \operatorname{Hypergeometric2F1} \left[ -\frac{1}{2}, m, 1+m, -\frac{bx}{a} \right] + b m x \operatorname{Hypergeometric2F1} \left[ \right. \right. \right. \right. \right. \\ \left. \left. \left. \left. \left. \frac{1}{2}, 1+m, 2+m, -\frac{bx}{a} \right] \right) \right) \right) \right) \right) / \left( a^3 (-2+m) (-1+m) m (1+m) \sqrt{a+bx} \right)$$

**Problem 1162: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{3-x} \sqrt{-2+x}} dx$$

Optimal (type 3, 8 leaves, 3 steps):

$$-\operatorname{ArcSin}[5-2x]$$

Result (type 3, 36 leaves):

$$\frac{2 \sqrt{-3+x} \sqrt{-2+x} \operatorname{ArcSinh}[\sqrt{-3+x}]}{\sqrt{-(-3+x)(-2+x)}}$$

**Problem 1170: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(6-3ex)^{1/4} (2+ex)^{3/4}} dx$$

Optimal (type 3, 241 leaves, 11 steps):

$$\frac{\sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (2-ex)^{1/4}}{(2+ex)^{1/4}}\right]}{3^{1/4} e} - \frac{\sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (2-ex)^{1/4}}{(2+ex)^{1/4}}\right]}{3^{1/4} e} -$$

$$\frac{\operatorname{Log}\left[\frac{\sqrt{6-3ex} - \sqrt{6} (2-ex)^{1/4} (2+ex)^{1/4} + \sqrt{3} \sqrt{2+ex}}{\sqrt{2+ex}}\right]}{\sqrt{2} 3^{1/4} e} + \frac{\operatorname{Log}\left[\frac{\sqrt{6-3ex} + \sqrt{6} (2-ex)^{1/4} (2+ex)^{1/4} + \sqrt{3} \sqrt{2+ex}}{\sqrt{2+ex}}\right]}{\sqrt{2} 3^{1/4} e}$$

Result (type 5, 43 leaves):

$$\frac{2\sqrt{2} (2+ex)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{4} (2+ex)\right]}{3^{1/4} e}$$

**Problem 1171: Result unnecessarily involves higher level functions.**

$$\int \frac{(a-iax)^{7/4}}{(a+iax)^{1/4}} dx$$

Optimal (type 4, 144 leaves, 6 steps):

$$\frac{14 a^2 x}{5 (a-iax)^{1/4} (a+iax)^{1/4}} - \frac{14}{15} i (a-iax)^{3/4} (a+iax)^{3/4} -$$

$$\frac{2 i (a-iax)^{7/4} (a+iax)^{3/4}}{5 a} - \frac{14 a^2 (1+x^2)^{1/4} \operatorname{EllipticE}\left[\frac{\operatorname{ArcTan}[x]}{2}, 2\right]}{5 (a-iax)^{1/4} (a+iax)^{1/4}}$$

Result (type 5, 84 leaves):

$$\frac{1}{15 (a+iax)^{1/4}} + 2 a (a-iax)^{3/4} \left(-10 i + 7 x - 3 i x^2 + 7 i 2^{3/4} (1+ix)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{ix}{2}\right]\right)$$

**Problem 1172: Result unnecessarily involves higher level functions.**

$$\int \frac{(a-iax)^{3/4}}{(a+iax)^{1/4}} dx$$

Optimal (type 4, 106 leaves, 5 steps):

$$\frac{2 a x}{(a-iax)^{1/4} (a+iax)^{1/4}} - \frac{2 i (a-iax)^{3/4} (a+iax)^{3/4}}{3 a} - \frac{2 a (1+x^2)^{1/4} \operatorname{EllipticE}\left[\frac{\operatorname{ArcTan}[x]}{2}, 2\right]}{(a-iax)^{1/4} (a+iax)^{1/4}}$$



Result (type 5, 74 leaves):

$$\frac{1}{3 (a + i a x)^{1/4}} 2 (a - i a x)^{3/4} \left( -i + x + i 2^{3/4} (1 + i x)^{1/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2} \right] \right)$$

**Problem 1173: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{1/4} (a + i a x)^{1/4}} dx$$

Optimal (type 4, 71 leaves, 4 steps):

$$\frac{2 x}{(a - i a x)^{1/4} (a + i a x)^{1/4}} - \frac{2 (1 + x^2)^{1/4} \text{EllipticE} \left[ \frac{\text{ArcTan}[x]}{2}, 2 \right]}{(a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 70 leaves):

$$\frac{2 i 2^{3/4} (1 + i x)^{1/4} (a - i a x)^{3/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2} \right]}{3 a (a + i a x)^{1/4}}$$

**Problem 1174: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{5/4} (a + i a x)^{1/4}} dx$$

Optimal (type 4, 78 leaves, 4 steps):

$$-\frac{2 i}{a (a - i a x)^{1/4} (a + i a x)^{1/4}} + \frac{2 (1 + x^2)^{1/4} \text{EllipticE} \left[ \frac{\text{ArcTan}[x]}{2}, 2 \right]}{a (a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 82 leaves):

$$\frac{\left( -6 i + 6 x - 2 \times 2^{3/4} (1 + i x)^{1/4} (i + x) \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2} \right] \right)}{\left( 3 a (a - i a x)^{1/4} (a + i a x)^{1/4} \right)}$$

**Problem 1175: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{9/4} (a + i a x)^{1/4}} dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$-\frac{4 i}{5 a (a - i a x)^{5/4} (a + i a x)^{1/4}} + \frac{2 (1 + x^2)^{1/4} \text{EllipticE} \left[ \frac{\text{ArcTan}[x]}{2}, 2 \right]}{5 a^2 (a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 97 leaves):

$$\left( 6 (2 + i x + x^2) - 2 \times 2^{3/4} (1 + i x)^{1/4} (i + x)^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right] \right) / \left( 15 a^2 (i + x) (a - i a x)^{1/4} (a + i a x)^{1/4} \right)$$

**Problem 1176: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{13/4} (a + i a x)^{1/4}} dx$$

Optimal (type 4, 115 leaves, 5 steps):

$$-\frac{4 i}{15 a^2 (a - i a x)^{5/4} (a + i a x)^{1/4}} - \frac{2 i (a + i a x)^{3/4}}{9 a^2 (a - i a x)^{9/4}} + \frac{2 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{15 a^3 (a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 103 leaves):

$$\left( 22 i - 4 x + 12 i x^2 + 6 x^3 - 2 \times 2^{3/4} (1 + i x)^{1/4} (i + x)^3 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right] \right) / \left( 45 a^3 (i + x)^2 (a - i a x)^{1/4} (a + i a x)^{1/4} \right)$$

**Problem 1177: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{17/4} (a + i a x)^{1/4}} dx$$

Optimal (type 4, 148 leaves, 6 steps):

$$-\frac{4 i}{39 a^3 (a - i a x)^{5/4} (a + i a x)^{1/4}} - \frac{2 i (a + i a x)^{3/4}}{13 a^2 (a - i a x)^{13/4}} - \frac{10 i (a + i a x)^{3/4}}{117 a^3 (a - i a x)^{9/4}} + \frac{2 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{39 a^4 (a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 102 leaves):

$$-\left( \left( 2 \left( 20 + 8 x^2 - 9 i x^3 - 3 x^4 + 2^{3/4} (1 + i x)^{1/4} (i + x)^4 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right] \right) \right) / \left( 117 a^4 (i + x)^3 (a - i a x)^{1/4} (a + i a x)^{1/4} \right) \right)$$

**Problem 1178: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{1/4}}{(a + i a x)^{1/4}} dx$$

Optimal (type 3, 256 leaves, 12 steps):

$$\begin{aligned}
 & - \frac{i (a - i a x)^{1/4} (a + i a x)^{3/4}}{a} - \frac{i \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2}} + \frac{i \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2}} \\
 & - \frac{i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{2 \sqrt{2}} + \frac{i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{2 \sqrt{2}}
 \end{aligned}$$

Result (type 5, 71 leaves):

$$\frac{1}{(a + i a x)^{1/4}} (a - i a x)^{1/4} \left( -i + x + i 2^{3/4} (1 + i x)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2}\right] \right)$$

**Problem 1179: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{3/4} (a + i a x)^{1/4}} dx$$

Optimal (type 3, 233 leaves, 11 steps):

$$\begin{aligned}
 & - \frac{i \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{a} + \frac{i \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{a} \\
 & - \frac{i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2} a} + \frac{i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2} a}
 \end{aligned}$$

Result (type 5, 68 leaves):

$$\frac{2 i 2^{3/4} (1 + i x)^{1/4} (a - i a x)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2}\right]}{a (a + i a x)^{1/4}}$$

**Problem 1184: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{3/4}}{(a + i a x)^{3/4}} dx$$

Optimal (type 3, 256 leaves, 12 steps):

$$\begin{aligned}
 & - \frac{i (a - i a x)^{3/4} (a + i a x)^{1/4}}{a} - \frac{3 i \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2}} + \frac{3 i \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2}} \\
 & - \frac{3 i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{2 \sqrt{2}} - \frac{3 i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{2 \sqrt{2}}
 \end{aligned}$$

Result (type 5, 71 leaves):

$$\frac{1}{(a + i a x)^{3/4}} (a - i a x)^{3/4} \left( -i + x + i 2^{1/4} (1 + i x)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right] \right)$$

**Problem 1185: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{1/4} (a + i a x)^{3/4}} dx$$

Optimal (type 3, 233 leaves, 11 steps):

$$-\frac{i \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{a} + \frac{i \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{a} +$$

$$\frac{i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2} a} - \frac{i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2} a}$$

Result (type 5, 70 leaves):

$$\frac{2 i 2^{1/4} (1 + i x)^{3/4} (a - i a x)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1 - i x}{2}\right]}{3 a (a + i a x)^{3/4}}$$

**Problem 1189: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{5/4}}{(a + i a x)^{3/4}} dx$$

Optimal (type 4, 112 leaves, 5 steps):

$$-\frac{10}{3} i (a - i a x)^{1/4} (a + i a x)^{1/4} -$$

$$\frac{2 i (a - i a x)^{5/4} (a + i a x)^{1/4}}{3 a} + \frac{10 a^2 (1 + x^2)^{3/4} \operatorname{EllipticF}\left[\frac{\operatorname{ArcTan}[x]}{2}, 2\right]}{3 (a - i a x)^{3/4} (a + i a x)^{3/4}}$$

Result (type 5, 80 leaves):

$$-\frac{1}{3 (a + i a x)^{3/4}}$$

$$+ \frac{2 i a (a - i a x)^{1/4} \left(6 + 5 i x + x^2 - 5 \times 2^{1/4} (1 + i x)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1 - i x}{2}\right]\right)}{3 (a + i a x)^{3/4}}$$

**Problem 1190: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{1/4}}{(a + i a x)^{3/4}} dx$$

Optimal (type 4, 76 leaves, 4 steps):

$$-\frac{2 i (a - i a x)^{1/4} (a + i a x)^{1/4}}{a} + \frac{2 a (1 + x^2)^{3/4} \operatorname{EllipticF}\left[\frac{\operatorname{ArcTan}[x]}{2}, 2\right]}{(a - i a x)^{3/4} (a + i a x)^{3/4}}$$

Result (type 5, 72 leaves):

$$\frac{1}{(a + i a x)^{3/4}} 2 (a - i a x)^{1/4} \left( -i + x + i 2^{1/4} (1 + i x)^{3/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2} \right] \right)$$

**Problem 1191: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{3/4} (a + i a x)^{3/4}} dx$$

Optimal (type 4, 43 leaves, 3 steps):

$$\frac{2 (1 + x^2)^{3/4} \text{EllipticF} \left[ \frac{\text{ArcTan}[x]}{2}, 2 \right]}{(a - i a x)^{3/4} (a + i a x)^{3/4}}$$

Result (type 5, 68 leaves):

$$\frac{2 i 2^{1/4} (1 + i x)^{3/4} (a - i a x)^{1/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2} \right]}{a (a + i a x)^{3/4}}$$

**Problem 1192: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{7/4} (a + i a x)^{3/4}} dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$-\frac{2 i (a + i a x)^{1/4}}{3 a^2 (a - i a x)^{3/4}} + \frac{2 (1 + x^2)^{3/4} \text{EllipticF} \left[ \frac{\text{ArcTan}[x]}{2}, 2 \right]}{3 a (a - i a x)^{3/4} (a + i a x)^{3/4}}$$

Result (type 5, 79 leaves):

$$\left( 2 \left( -i + x + 2^{1/4} (1 + i x)^{3/4} (i + x) \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2} \right] \right) \right) / \left( 3 a (a - i a x)^{3/4} (a + i a x)^{3/4} \right)$$

**Problem 1193: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{11/4} (a + i a x)^{3/4}} dx$$

Optimal (type 4, 115 leaves, 5 steps):

$$-\frac{2 i (a + i a x)^{1/4}}{7 a^2 (a - i a x)^{7/4}} - \frac{2 i (a + i a x)^{1/4}}{7 a^3 (a - i a x)^{3/4}} + \frac{2 (1 + x^2)^{3/4} \text{EllipticF} \left[ \frac{\text{ArcTan}[x]}{2}, 2 \right]}{7 a^2 (a - i a x)^{3/4} (a + i a x)^{3/4}}$$

Result (type 5, 93 leaves):

$$\left( 2 \left( 2 + i x + x^2 + 2^{1/4} (1 + i x)^{3/4} (i + x)^2 \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2} \right] \right) \right) / \left( 7 a^2 (i + x) (a - i a x)^{3/4} (a + i a x)^{3/4} \right)$$

**Problem 1194: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{7/4}}{(a + i a x)^{7/4}} dx$$

Optimal (type 3, 291 leaves, 13 steps):

$$\frac{4 i (a - i a x)^{7/4}}{3 a (a + i a x)^{3/4}} + \frac{7 i (a - i a x)^{3/4} (a + i a x)^{1/4}}{3 a} + \frac{7 i \text{ArcTan} \left[ 1 - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}} \right]}{\sqrt{2}} - \frac{7 i \text{ArcTan} \left[ 1 + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}} \right]}{\sqrt{2}} - \frac{7 i \text{Log} \left[ 1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}} \right]}{2 \sqrt{2}} + \frac{7 i \text{Log} \left[ 1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}} \right]}{2 \sqrt{2}}$$

Result (type 5, 76 leaves):

$$\frac{1}{3 (a + i a x)^{3/4}} (a - i a x)^{3/4} \left( 11 i - 3 x - 7 i 2^{1/4} (1 + i x)^{3/4} \text{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2} \right] \right)$$

**Problem 1195: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{3/4}}{(a + i a x)^{7/4}} dx$$

Optimal (type 3, 266 leaves, 12 steps):

$$\frac{4 i (a - i a x)^{3/4}}{3 a (a + i a x)^{3/4}} + \frac{i \sqrt{2} \text{ArcTan} \left[ 1 - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}} \right]}{a} - \frac{i \sqrt{2} \text{ArcTan} \left[ 1 + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}} \right]}{a} - \frac{i \text{Log} \left[ 1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}} \right]}{\sqrt{2} a} + \frac{i \text{Log} \left[ 1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}} \right]}{\sqrt{2} a}$$

Result (type 5, 73 leaves):

$$-\frac{1}{3 a (a + i a x)^{3/4}} 2 i (a - i a x)^{3/4} \left( -2 + 2^{1/4} (1 + i x)^{3/4} \text{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2} \right] \right)$$

**Problem 1199: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{9/4}}{(a + i a x)^{7/4}} dx$$

Optimal (type 4, 139 leaves, 6 steps):

$$\frac{4 i (a - i a x)^{9/4}}{3 a (a + i a x)^{3/4}} + 10 i (a - i a x)^{1/4} (a + i a x)^{1/4} + \frac{2 i (a - i a x)^{5/4} (a + i a x)^{1/4}}{a} - \frac{10 a^2 (1 + x^2)^{3/4} \text{EllipticF}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{(a - i a x)^{3/4} (a + i a x)^{3/4}}$$

Result (type 5, 80 leaves):

$$\frac{1}{3 (a + i a x)^{3/4}} + 2 i a (a - i a x)^{1/4} \left( 20 + 11 i x + x^2 - 15 \times 2^{1/4} (1 + i x)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2}\right] \right)$$

**Problem 1200: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{5/4}}{(a + i a x)^{7/4}} dx$$

Optimal (type 4, 113 leaves, 5 steps):

$$\frac{4 i (a - i a x)^{5/4}}{3 a (a + i a x)^{3/4}} + \frac{10 i (a - i a x)^{1/4} (a + i a x)^{1/4}}{3 a} - \frac{10 a (1 + x^2)^{3/4} \text{EllipticF}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{3 (a - i a x)^{3/4} (a + i a x)^{3/4}}$$

Result (type 5, 76 leaves):

$$-\frac{1}{3 (a + i a x)^{3/4}} + 2 (a - i a x)^{1/4} \left( -7 i + 3 x + 5 i 2^{1/4} (1 + i x)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2}\right] \right)$$

**Problem 1201: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{1/4}}{(a + i a x)^{7/4}} dx$$

Optimal (type 4, 79 leaves, 4 steps):

$$\frac{4 i (a - i a x)^{1/4}}{3 a (a + i a x)^{3/4}} - \frac{2 (1 + x^2)^{3/4} \text{EllipticF}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{3 (a - i a x)^{3/4} (a + i a x)^{3/4}}$$

Result (type 5, 73 leaves):

$$-\frac{1}{3 a (a+i a x)^{3/4}} 2 i (a-i a x)^{1/4} \left( -2 + 2^{1/4} (1+i x)^{3/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2} \right] \right)$$

**Problem 1202: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a-i a x)^{3/4} (a+i a x)^{7/4}} dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$\frac{2 i (a-i a x)^{1/4}}{3 a^2 (a+i a x)^{3/4}} + \frac{2 (1+x^2)^{3/4} \text{EllipticF} \left[ \frac{\text{ArcTan}[x]}{2}, 2 \right]}{3 a (a-i a x)^{3/4} (a+i a x)^{3/4}}$$

Result (type 5, 73 leaves):

$$\frac{1}{3 a^2 (a+i a x)^{3/4}} 2 i (a-i a x)^{1/4} \left( 1 + 2^{1/4} (1+i x)^{3/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2} \right] \right)$$

**Problem 1203: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a-i a x)^{7/4} (a+i a x)^{7/4}} dx$$

Optimal (type 4, 81 leaves, 4 steps):

$$\frac{2 x}{3 a^2 (a-i a x)^{3/4} (a+i a x)^{3/4}} + \frac{2 (1+x^2)^{3/4} \text{EllipticF} \left[ \frac{\text{ArcTan}[x]}{2}, 2 \right]}{3 a^2 (a-i a x)^{3/4} (a+i a x)^{3/4}}$$

Result (type 5, 76 leaves):

$$\frac{2 \left( x + 2^{1/4} (1+i x)^{3/4} (i+x) \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2} \right] \right)}{3 a^2 (a-i a x)^{3/4} (a+i a x)^{3/4}}$$

**Problem 1204: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a-i a x)^{11/4} (a+i a x)^{7/4}} dx$$

Optimal (type 4, 114 leaves, 5 steps):

$$-\frac{2 i}{7 a^2 (a-i a x)^{7/4} (a+i a x)^{3/4}} + \frac{10 x}{21 a^3 (a-i a x)^{3/4} (a+i a x)^{3/4}} + \frac{10 (1+x^2)^{3/4} \text{EllipticF} \left[ \frac{\text{ArcTan}[x]}{2}, 2 \right]}{21 a^3 (a-i a x)^{3/4} (a+i a x)^{3/4}}$$

Result (type 5, 96 leaves):



$$\left( 2 \left( 3 + 5 i x + 5 x^2 + 5 \times 2^{1/4} (1 + i x)^{3/4} (i + x)^2 \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2} \right] \right) \right) / \left( 21 a^3 (i + x) (a - i a x)^{3/4} (a + i a x)^{3/4} \right)$$

**Problem 1205: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{15/4} (a + i a x)^{7/4}} dx$$

Optimal (type 4, 147 leaves, 6 steps):

$$-\frac{2 i}{11 a^2 (a - i a x)^{11/4} (a + i a x)^{3/4}} - \frac{2 i}{11 a^3 (a - i a x)^{7/4} (a + i a x)^{3/4}} + \frac{10 x}{33 a^4 (a - i a x)^{3/4} (a + i a x)^{3/4}} + \frac{10 (1 + x^2)^{3/4} \text{EllipticF} \left[ \frac{\text{ArcTan}[x]}{2}, 2 \right]}{33 a^4 (a - i a x)^{3/4} (a + i a x)^{3/4}}$$

Result (type 5, 103 leaves):

$$\left( 2 \left( 6 i - 2 x + 10 i x^2 + 5 x^3 + 5 \times 2^{1/4} (1 + i x)^{3/4} (i + x)^3 \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2} \right] \right) \right) / \left( 33 a^4 (i + x)^2 (a - i a x)^{3/4} (a + i a x)^{3/4} \right)$$

**Problem 1206: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{7/4}}{(a + i a x)^{5/4}} dx$$

Optimal (type 4, 137 leaves, 6 steps):

$$-\frac{14 a x}{(a - i a x)^{1/4} (a + i a x)^{1/4}} + \frac{4 i (a - i a x)^{7/4}}{a (a + i a x)^{1/4}} + \frac{14 i (a - i a x)^{3/4} (a + i a x)^{3/4}}{3 a} + \frac{14 a (1 + x^2)^{1/4} \text{EllipticE} \left[ \frac{\text{ArcTan}[x]}{2}, 2 \right]}{(a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 74 leaves):

$$-\frac{1}{3 (a + i a x)^{1/4}} + 2 (a - i a x)^{3/4} \left( -13 i + x + 7 i 2^{3/4} (1 + i x)^{1/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2} \right] \right)$$

**Problem 1207: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{3/4}}{(a + i a x)^{5/4}} dx$$

Optimal (type 4, 102 leaves, 5 steps):

$$-\frac{6x}{(a-ix)^{1/4}(a+ix)^{1/4}} + \frac{4i(a-ix)^{3/4}}{a(a+ix)^{1/4}} + \frac{6(1+x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{(a-ix)^{1/4}(a+ix)^{1/4}}$$

Result (type 5, 71 leaves):

$$-\frac{1}{a(a+ix)^{1/4}} 2i(a-ix)^{3/4} \left(-2 + 2^{3/4}(1+ix)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{ix}{2}\right]\right)$$

**Problem 1208: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a-ix)^{1/4}(a+ix)^{5/4}} dx$$

Optimal (type 4, 78 leaves, 4 steps):

$$\frac{2i}{a(a-ix)^{1/4}(a+ix)^{1/4}} + \frac{2(1+x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{a(a-ix)^{1/4}(a+ix)^{1/4}}$$

Result (type 5, 73 leaves):

$$-\frac{1}{3a^2(a+ix)^{1/4}} 2i(a-ix)^{3/4} \left(-3 + 2^{3/4}(1+ix)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{ix}{2}\right]\right)$$

**Problem 1209: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a-ix)^{5/4}(a+ix)^{5/4}} dx$$

Optimal (type 4, 46 leaves, 3 steps):

$$\frac{2(1+x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{a^2(a-ix)^{1/4}(a+ix)^{1/4}}$$

Result (type 5, 79 leaves):

$$\frac{6x - 2 \times 2^{3/4}(1+ix)^{1/4}(ix) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{ix}{2}\right]}{3a^2(a-ix)^{1/4}(a+ix)^{1/4}}$$

**Problem 1210: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a-ix)^{9/4}(a+ix)^{5/4}} dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$-\frac{2i}{5a^2(a-ix)^{5/4}(a+ix)^{1/4}} + \frac{6(1+x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{5a^3(a-ix)^{1/4}(a+ix)^{1/4}}$$

Result (type 5, 96 leaves):

$$\frac{\left(2 + 6 i x + 6 x^2 - 2 \times 2^{3/4} (1 + i x)^{1/4} (i + x)^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)}{\left(5 a^3 (i + x) (a - i a x)^{1/4} (a + i a x)^{1/4}\right)}$$

**Problem 1211: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{13/4} (a + i a x)^{5/4}} dx$$

Optimal (type 4, 115 leaves, 5 steps):

$$-\frac{2 i}{9 a^2 (a - i a x)^{9/4} (a + i a x)^{1/4}} - \frac{2 i}{9 a^3 (a - i a x)^{5/4} (a + i a x)^{1/4}} + \frac{2 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{3 a^4 (a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 103 leaves):

$$\frac{\left(4 i - 4 x + 12 i x^2 + 6 x^3 - 2 \times 2^{3/4} (1 + i x)^{1/4} (i + x)^3 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)}{\left(9 a^4 (i + x)^2 (a - i a x)^{1/4} (a + i a x)^{1/4}\right)}$$

**Problem 1212: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{5/4}}{(a + i a x)^{5/4}} dx$$

Optimal (type 3, 287 leaves, 13 steps):

$$\frac{4 i (a - i a x)^{5/4}}{a (a + i a x)^{1/4}} + \frac{5 i (a - i a x)^{1/4} (a + i a x)^{3/4}}{a} + \frac{5 i \text{ArcTan}\left[1 - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2}} - \frac{5 i \text{ArcTan}\left[1 + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2}} + \frac{5 i \text{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{2 \sqrt{2}} - \frac{5 i \text{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{2 \sqrt{2}}$$

Result (type 5, 72 leaves):

$$-\frac{1}{(a + i a x)^{1/4}} (a - i a x)^{1/4} \left(-9 i + x + 5 i 2^{3/4} (1 + i x)^{1/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)$$

**Problem 1213: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{1/4}}{(a + i a x)^{5/4}} dx$$

Optimal (type 3, 264 leaves, 12 steps):

$$\frac{4 i (a - i a x)^{1/4}}{a (a + i a x)^{1/4}} + \frac{i \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{a} - \frac{i \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{a} +$$

$$\frac{i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} - \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2} a} - \frac{i \operatorname{Log}\left[1 + \frac{\sqrt{a - i a x}}{\sqrt{a + i a x}} + \frac{\sqrt{2} (a - i a x)^{1/4}}{(a + i a x)^{1/4}}\right]}{\sqrt{2} a}$$

Result (type 5, 71 leaves):

$$-\frac{1}{a (a + i a x)^{1/4}} 2 i (a - i a x)^{1/4} \left(-2 + 2^{3/4} (1 + i x)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)$$

**Problem 1217: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{7/4}}{(a + i a x)^{9/4}} dx$$

Optimal (type 4, 141 leaves, 6 steps):

$$\frac{4 i (a - i a x)^{7/4}}{5 a (a + i a x)^{5/4}} + \frac{42 x}{5 (a - i a x)^{1/4} (a + i a x)^{1/4}} -$$

$$\frac{28 i (a - i a x)^{3/4}}{5 a (a + i a x)^{1/4}} - \frac{42 (1 + x^2)^{1/4} \operatorname{EllipticE}\left[\frac{\operatorname{ArcTan}[x]}{2}, 2\right]}{5 (a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 84 leaves):

$$\left(2 (a - i a x)^{3/4} \left(-12 - 16 i x + 7 \times 2^{3/4} (1 + i x)^{5/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)\right) /$$

$$\left(5 a (-i + x) (a + i a x)^{1/4}\right)$$

**Problem 1218: Result unnecessarily involves higher level functions.**

$$\int \frac{(a - i a x)^{3/4}}{(a + i a x)^{9/4}} dx$$

Optimal (type 4, 115 leaves, 5 steps):

$$\frac{4 i (a - i a x)^{3/4}}{5 a (a + i a x)^{5/4}} - \frac{6 i}{5 a (a - i a x)^{1/4} (a + i a x)^{1/4}} - \frac{6 (1 + x^2)^{1/4} \operatorname{EllipticE}\left[\frac{\operatorname{ArcTan}[x]}{2}, 2\right]}{5 a (a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 83 leaves):

$$\left(2 (a - i a x)^{3/4} \left(-1 - 3 i x + 2^{3/4} (1 + i x)^{5/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)\right) /$$

$$\left(5 a^2 (-i + x) (a + i a x)^{1/4}\right)$$

**Problem 1219: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{1/4} (a + i a x)^{9/4}} dx$$

Optimal (type 4, 82 leaves, 4 steps):

$$\frac{4 i}{5 a (a - i a x)^{1/4} (a + i a x)^{5/4}} + \frac{2 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{5 a^2 (a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 84 leaves):

$$\frac{\left(2 (a - i a x)^{3/4} \left(6 + 3 i x - 2^{3/4} (1 + i x)^{5/4} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)\right)}{\left(15 a^3 (-i + x) (a + i a x)^{1/4}\right)}$$

**Problem 1220: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{5/4} (a + i a x)^{9/4}} dx$$

Optimal (type 4, 82 leaves, 5 steps):

$$\frac{2 i}{5 a^2 (a - i a x)^{1/4} (a + i a x)^{5/4}} + \frac{6 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{5 a^3 (a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 94 leaves):

$$\frac{\left(2 - 6 i x + 6 x^2 - 2 \times 2^{3/4} (1 + i x)^{1/4} (1 + x^2) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)}{\left(5 a^3 (-i + x) (a - i a x)^{1/4} (a + i a x)^{1/4}\right)}$$

**Problem 1221: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a - i a x)^{9/4} (a + i a x)^{9/4}} dx$$

Optimal (type 4, 88 leaves, 4 steps):

$$\frac{2 x}{5 a^4 (a - i a x)^{1/4} (a + i a x)^{1/4} (1 + x^2)} + \frac{6 (1 + x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{5 a^4 (a - i a x)^{1/4} (a + i a x)^{1/4}}$$

Result (type 5, 98 leaves):

$$\frac{\left(8 x + 6 x^3 - 2 \times 2^{3/4} (1 + i x)^{1/4} (-i + x) (i + x)^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)}{\left(5 a^4 (a - i a x)^{1/4} (a + i a x)^{1/4} (1 + x^2)\right)}$$

### Problem 1222: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - i a x)^{13/4} (a + i a x)^{9/4}} dx$$

Optimal (type 4, 121 leaves, 5 steps):

$$-\frac{2i}{9a^2(a - i a x)^{9/4}(a + i a x)^{5/4}} + \frac{14x}{45a^5(a - i a x)^{1/4}(a + i a x)^{1/4}(1+x^2)} + \frac{14(1+x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{15a^5(a - i a x)^{1/4}(a + i a x)^{1/4}}$$

Result (type 5, 120 leaves):

$$\left(2 \left(5 + 28i x + 28x^2 + 21i x^3 + 21x^4 - 7 \times 2^{3/4} (1 + i x)^{1/4} (-i + x) (i + x)^3 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)\right) / \left(45a^5(-i + x)(i + x)^2(a - i a x)^{1/4}(a + i a x)^{1/4}\right)$$

### Problem 1223: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a - i a x)^{17/4} (a + i a x)^{9/4}} dx$$

Optimal (type 4, 154 leaves, 6 steps):

$$-\frac{2i}{13a^2(a - i a x)^{13/4}(a + i a x)^{5/4}} - \frac{2i}{13a^3(a - i a x)^{9/4}(a + i a x)^{5/4}} + \frac{14x}{65a^6(a - i a x)^{1/4}(a + i a x)^{1/4}(1+x^2)} + \frac{42(1+x^2)^{1/4} \text{EllipticE}\left[\frac{\text{ArcTan}[x]}{2}, 2\right]}{65a^6(a - i a x)^{1/4}(a + i a x)^{1/4}}$$

Result (type 5, 127 leaves):

$$\left(2 \left(10i - 23x + 56i x^2 + 7x^3 + 42i x^4 + 21x^5 - 7 \times 2^{3/4} (1 + i x)^{1/4} (-i + x) (i + x)^4 \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{1}{2} - \frac{i x}{2}\right]\right)\right) / \left(65a^6(-i + x)(i + x)^3(a - i a x)^{1/4}(a + i a x)^{1/4}\right)$$

### Problem 1224: Result unnecessarily involves higher level functions.

$$\int \frac{(a - i a x)^{5/4}}{(a + i a x)^{9/4}} dx$$

Optimal (type 3, 297 leaves, 13 steps):

$$\frac{4i(a-iax)^{5/4}}{5a(a+iax)^{5/4}} - \frac{4i(a-iax)^{1/4}}{a(a+iax)^{1/4}} -$$

$$\frac{i\sqrt{2}\operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(a-iax)^{1/4}}{(a+iax)^{1/4}}\right]}{a} + \frac{i\sqrt{2}\operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(a-iax)^{1/4}}{(a+iax)^{1/4}}\right]}{a} -$$

$$\frac{i\operatorname{Log}\left[1 + \frac{\sqrt{a-iax}}{\sqrt{a+iax}} - \frac{\sqrt{2}(a-iax)^{1/4}}{(a+iax)^{1/4}}\right]}{\sqrt{2}a} + \frac{i\operatorname{Log}\left[1 + \frac{\sqrt{a-iax}}{\sqrt{a+iax}} + \frac{\sqrt{2}(a-iax)^{1/4}}{(a+iax)^{1/4}}\right]}{\sqrt{2}a}$$

Result (type 5, 84 leaves):

$$\left(2(a-iax)^{1/4} \left(-8 - 12ix + 5 \times 2^{3/4} (1+ix)^{5/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2} - \frac{ix}{2}\right]\right)\right) /$$

$$(5a(-i+x)(a+iax)^{1/4})$$

**Problem 1235: Result more than twice size of optimal antiderivative.**

$$\int (3-6x)^m (2+4x)^m dx$$

Optimal (type 5, 20 leaves, 2 steps):

$$6^m x \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -m, \frac{3}{2}, 4x^2\right]$$

Result (type 5, 42 leaves):

$$(3-6x)^m x (2+4x)^m (1-4x^2)^{-m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -m, \frac{3}{2}, 4x^2\right]$$

**Problem 1236: Result more than twice size of optimal antiderivative.**

$$\int (a+bx)^4 (c+dx) dx$$

Optimal (type 1, 38 leaves, 2 steps):

$$\frac{(bc-ad)(a+bx)^5}{5b^2} + \frac{d(a+bx)^6}{6b^2}$$

Result (type 1, 84 leaves):

$$\frac{1}{30} x (15a^4(2c+dx) + 20a^3bx(3c+2dx) +$$

$$15a^2b^2x^2(4c+3dx) + 6ab^3x^3(5c+4dx) + b^4x^4(6c+5dx))$$

**Problem 1246: Result more than twice size of optimal antiderivative.**

$$\int (a+bx)^4 (c+dx)^2 dx$$

Optimal (type 1, 65 leaves, 2 steps):

$$\frac{(bc - ad)^2 (a + bx)^5}{5b^3} + \frac{d(bc - ad)(a + bx)^6}{3b^3} + \frac{d^2(a + bx)^7}{7b^3}$$

Result (type 1, 148 leaves):

$$a^4 c^2 x + a^3 c (2bc + ad) x^2 + \frac{1}{3} a^2 (6b^2 c^2 + 8abcd + a^2 d^2) x^3 + ab (b^2 c^2 + 3abcd + a^2 d^2) x^4 + \frac{1}{5} b^2 (b^2 c^2 + 8abcd + 6a^2 d^2) x^5 + \frac{1}{3} b^3 d (bc + 2ad) x^6 + \frac{1}{7} b^4 d^2 x^7$$

**Problem 1258: Result more than twice size of optimal antiderivative.**

$$\int (a + bx)^5 (c + dx)^3 dx$$

Optimal (type 1, 92 leaves, 2 steps):

$$\frac{(bc - ad)^3 (a + bx)^6}{6b^4} + \frac{3d(bc - ad)^2 (a + bx)^7}{7b^4} + \frac{3d^2(bc - ad)(a + bx)^8}{8b^4} + \frac{d^3(a + bx)^9}{9b^4}$$

Result (type 1, 235 leaves):

$$\frac{1}{504} x (126a^5 (4c^3 + 6c^2 dx + 4cd^2 x^2 + d^3 x^3) + 126a^4 bx (10c^3 + 20c^2 dx + 15cd^2 x^2 + 4d^3 x^3) + 84a^3 b^2 x^2 (20c^3 + 45c^2 dx + 36cd^2 x^2 + 10d^3 x^3) + 36a^2 b^3 x^3 (35c^3 + 84c^2 dx + 70cd^2 x^2 + 20d^3 x^3) + 9a b^4 x^4 (56c^3 + 140c^2 dx + 120cd^2 x^2 + 35d^3 x^3) + b^5 x^5 (84c^3 + 216c^2 dx + 189cd^2 x^2 + 56d^3 x^3))$$

**Problem 1259: Result more than twice size of optimal antiderivative.**

$$\int (a + bx)^4 (c + dx)^3 dx$$

Optimal (type 1, 92 leaves, 2 steps):

$$\frac{(bc - ad)^3 (a + bx)^5}{5b^4} + \frac{d(bc - ad)^2 (a + bx)^6}{2b^4} + \frac{3d^2(bc - ad)(a + bx)^7}{7b^4} + \frac{d^3(a + bx)^8}{8b^4}$$

Result (type 1, 217 leaves):

$$a^4 c^3 x + \frac{1}{2} a^3 c^2 (4bc + 3ad) x^2 + a^2 c (2b^2 c^2 + 4abcd + a^2 d^2) x^3 + \frac{1}{4} a (4b^3 c^3 + 18a b^2 c^2 d + 12a^2 b c d^2 + a^3 d^3) x^4 + \frac{1}{5} b (b^3 c^3 + 12a b^2 c^2 d + 18a^2 b c d^2 + 4a^3 d^3) x^5 + \frac{1}{2} b^2 d (b^2 c^2 + 4abcd + 2a^2 d^2) x^6 + \frac{1}{7} b^3 d^2 (3bc + 4ad) x^7 + \frac{1}{8} b^4 d^3 x^8$$

**Problem 1268: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + dx)^3}{(a + bx)^5} dx$$



Optimal (type 1, 28 leaves, 1 step):

$$-\frac{(c+dx)^4}{4(bc-ad)(a+bx)^4}$$

Result (type 1, 91 leaves):

$$-\frac{1}{4b^4(a+bx)^4} (a^3d^3 + a^2bd^2(c+4dx) + ab^2d(c^2 + 4cdx + 6d^2x^2) + b^3(c^3 + 4c^2dx + 6cd^2x^2 + 4d^3x^3))$$

**Problem 1273: Result more than twice size of optimal antiderivative.**

$$\int (a+bx)^9 (c+dx)^7 dx$$

Optimal (type 1, 200 leaves, 2 steps):

$$\begin{aligned} & \frac{(bc-ad)^7 (a+bx)^{10}}{10b^8} + \frac{7d(bc-ad)^6 (a+bx)^{11}}{11b^8} + \\ & \frac{7d^2(bc-ad)^5 (a+bx)^{12}}{4b^8} + \frac{35d^3(bc-ad)^4 (a+bx)^{13}}{13b^8} + \frac{5d^4(bc-ad)^3 (a+bx)^{14}}{2b^8} + \\ & \frac{7d^5(bc-ad)^2 (a+bx)^{15}}{5b^8} + \frac{7d^6(bc-ad)(a+bx)^{16}}{16b^8} + \frac{d^7(a+bx)^{17}}{17b^8} \end{aligned}$$

Result (type 1, 993 leaves):

$$\begin{aligned}
& a^9 c^7 x + \frac{1}{2} a^8 c^6 (9 b c + 7 a d) x^2 + a^7 c^5 (12 b^2 c^2 + 21 a b c d + 7 a^2 d^2) x^3 + \\
& \frac{7}{4} a^6 c^4 (12 b^3 c^3 + 36 a b^2 c^2 d + 27 a^2 b c d^2 + 5 a^3 d^3) x^4 + \\
& \frac{7}{5} a^5 c^3 (18 b^4 c^4 + 84 a b^3 c^3 d + 108 a^2 b^2 c^2 d^2 + 45 a^3 b c d^3 + 5 a^4 d^4) x^5 + \\
& \frac{7}{2} a^4 c^2 (6 b^5 c^5 + 42 a b^4 c^4 d + 84 a^2 b^3 c^3 d^2 + 60 a^3 b^2 c^2 d^3 + 15 a^4 b c d^4 + a^5 d^5) x^6 + \\
& a^3 c (12 b^6 c^6 + 126 a b^5 c^5 d + 378 a^2 b^4 c^4 d^2 + 420 a^3 b^3 c^3 d^3 + 180 a^4 b^2 c^2 d^4 + 27 a^5 b c d^5 + a^6 d^6) x^7 + \\
& \frac{1}{8} a^2 (36 b^7 c^7 + 588 a b^6 c^6 d + 2646 a^2 b^5 c^5 d^2 + \\
& \quad 4410 a^3 b^4 c^4 d^3 + 2940 a^4 b^3 c^3 d^4 + 756 a^5 b^2 c^2 d^5 + 63 a^6 b c d^6 + a^7 d^7) x^8 + \\
& a b (b^7 c^7 + 28 a b^6 c^6 d + 196 a^2 b^5 c^5 d^2 + 490 a^3 b^4 c^4 d^3 + 490 a^4 b^3 c^3 d^4 + 196 a^5 b^2 c^2 d^5 + \\
& \quad 28 a^6 b c d^6 + a^7 d^7) x^9 + \frac{1}{10} b^2 (b^7 c^7 + 63 a b^6 c^6 d + 756 a^2 b^5 c^5 d^2 + 2940 a^3 b^4 c^4 d^3 + \\
& \quad 4410 a^4 b^3 c^3 d^4 + 2646 a^5 b^2 c^2 d^5 + 588 a^6 b c d^6 + 36 a^7 d^7) x^{10} + \frac{7}{11} b^3 d \\
& \quad (b^6 c^6 + 27 a b^5 c^5 d + 180 a^2 b^4 c^4 d^2 + 420 a^3 b^3 c^3 d^3 + 378 a^4 b^2 c^2 d^4 + 126 a^5 b c d^5 + 12 a^6 d^6) x^{11} + \\
& \frac{7}{4} b^4 d^2 (b^5 c^5 + 15 a b^4 c^4 d + 60 a^2 b^3 c^3 d^2 + 84 a^3 b^2 c^2 d^3 + 42 a^4 b c d^4 + 6 a^5 d^5) x^{12} + \\
& \frac{7}{13} b^5 d^3 (5 b^4 c^4 + 45 a b^3 c^3 d + 108 a^2 b^2 c^2 d^2 + 84 a^3 b c d^3 + 18 a^4 d^4) x^{13} + \\
& \frac{1}{2} b^6 d^4 (5 b^3 c^3 + 27 a b^2 c^2 d + 36 a^2 b c d^2 + 12 a^3 d^3) x^{14} + \\
& \frac{1}{5} b^7 d^5 (7 b^2 c^2 + 21 a b c d + 12 a^2 d^2) x^{15} + \frac{1}{16} b^8 d^6 (7 b c + 9 a d) x^{16} + \frac{1}{17} b^9 d^7 x^{17}
\end{aligned}$$

**Problem 1274: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^8 (c + d x)^7 dx$$

Optimal (type 1, 200 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b c - a d)^7 (a + b x)^9}{9 b^8} + \frac{7 d (b c - a d)^6 (a + b x)^{10}}{10 b^8} + \\
& \frac{21 d^2 (b c - a d)^5 (a + b x)^{11}}{11 b^8} + \frac{35 d^3 (b c - a d)^4 (a + b x)^{12}}{12 b^8} + \frac{35 d^4 (b c - a d)^3 (a + b x)^{13}}{13 b^8} + \\
& \frac{3 d^5 (b c - a d)^2 (a + b x)^{14}}{2 b^8} + \frac{7 d^6 (b c - a d) (a + b x)^{15}}{15 b^8} + \frac{d^7 (a + b x)^{16}}{16 b^8}
\end{aligned}$$

Result (type 1, 897 leaves):

$$\begin{aligned}
 & a^8 c^7 x + \frac{1}{2} a^7 c^6 (8 b c + 7 a d) x^2 + \frac{7}{3} a^6 c^5 (4 b^2 c^2 + 8 a b c d + 3 a^2 d^2) x^3 + \\
 & \frac{7}{4} a^5 c^4 (8 b^3 c^3 + 28 a b^2 c^2 d + 24 a^2 b c d^2 + 5 a^3 d^3) x^4 + \\
 & \frac{7}{5} a^4 c^3 (10 b^4 c^4 + 56 a b^3 c^3 d + 84 a^2 b^2 c^2 d^2 + 40 a^3 b c d^3 + 5 a^4 d^4) x^5 + \\
 & \frac{7}{6} a^3 c^2 (8 b^5 c^5 + 70 a b^4 c^4 d + 168 a^2 b^3 c^3 d^2 + 140 a^3 b^2 c^2 d^3 + 40 a^4 b c d^4 + 3 a^5 d^5) x^6 + \\
 & a^2 c (4 b^6 c^6 + 56 a b^5 c^5 d + 210 a^2 b^4 c^4 d^2 + 280 a^3 b^3 c^3 d^3 + 140 a^4 b^2 c^2 d^4 + 24 a^5 b c d^5 + a^6 d^6) x^7 + \\
 & \frac{1}{8} a (8 b^7 c^7 + 196 a b^6 c^6 d + 1176 a^2 b^5 c^5 d^2 + 2450 a^3 b^4 c^4 d^3 + 1960 a^4 b^3 c^3 d^4 + \\
 & \quad 588 a^5 b^2 c^2 d^5 + 56 a^6 b c d^6 + a^7 d^7) x^8 + \frac{1}{9} b (b^7 c^7 + 56 a b^6 c^6 d + 588 a^2 b^5 c^5 d^2 + \\
 & \quad 1960 a^3 b^4 c^4 d^3 + 2450 a^4 b^3 c^3 d^4 + 1176 a^5 b^2 c^2 d^5 + 196 a^6 b c d^6 + 8 a^7 d^7) x^9 + \\
 & \frac{7}{10} b^2 d (b^6 c^6 + 24 a b^5 c^5 d + 140 a^2 b^4 c^4 d^2 + 280 a^3 b^3 c^3 d^3 + 210 a^4 b^2 c^2 d^4 + 56 a^5 b c d^5 + 4 a^6 d^6) x^{10} + \\
 & \frac{7}{11} b^3 d^2 (3 b^5 c^5 + 40 a b^4 c^4 d + 140 a^2 b^3 c^3 d^2 + 168 a^3 b^2 c^2 d^3 + 70 a^4 b c d^4 + 8 a^5 d^5) x^{11} + \\
 & \frac{7}{12} b^4 d^3 (5 b^4 c^4 + 40 a b^3 c^3 d + 84 a^2 b^2 c^2 d^2 + 56 a^3 b c d^3 + 10 a^4 d^4) x^{12} + \\
 & \frac{7}{13} b^5 d^4 (5 b^3 c^3 + 24 a b^2 c^2 d + 28 a^2 b c d^2 + 8 a^3 d^3) x^{13} + \\
 & \frac{1}{2} b^6 d^5 (3 b^2 c^2 + 8 a b c d + 4 a^2 d^2) x^{14} + \frac{1}{15} b^7 d^6 (7 b c + 8 a d) x^{15} + \frac{1}{16} b^8 d^7 x^{16}
 \end{aligned}$$

**Problem 1275: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^7 (c + d x)^7 dx$$

Optimal (type 1, 200 leaves, 2 steps):

$$\begin{aligned}
 & \frac{(b c - a d)^7 (a + b x)^8}{8 b^8} + \frac{7 d (b c - a d)^6 (a + b x)^9}{9 b^8} + \frac{21 d^2 (b c - a d)^5 (a + b x)^{10}}{10 b^8} + \\
 & \frac{35 d^3 (b c - a d)^4 (a + b x)^{11}}{11 b^8} + \frac{35 d^4 (b c - a d)^3 (a + b x)^{12}}{12 b^8} + \\
 & \frac{21 d^5 (b c - a d)^2 (a + b x)^{13}}{13 b^8} + \frac{d^6 (b c - a d) (a + b x)^{14}}{2 b^8} + \frac{d^7 (a + b x)^{15}}{15 b^8}
 \end{aligned}$$

Result (type 1, 785 leaves):

$$\begin{aligned}
 & a^7 c^7 x + \frac{7}{2} a^6 c^6 (b c + a d) x^2 + \frac{7}{3} a^5 c^5 (3 b^2 c^2 + 7 a b c d + 3 a^2 d^2) x^3 + \\
 & \frac{7}{4} a^4 c^4 (5 b^3 c^3 + 21 a b^2 c^2 d + 21 a^2 b c d^2 + 5 a^3 d^3) x^4 + \\
 & \frac{7}{5} a^3 c^3 (5 b^4 c^4 + 35 a b^3 c^3 d + 63 a^2 b^2 c^2 d^2 + 35 a^3 b c d^3 + 5 a^4 d^4) x^5 + \\
 & \frac{7}{6} a^2 c^2 (3 b^5 c^5 + 35 a b^4 c^4 d + 105 a^2 b^3 c^3 d^2 + 105 a^3 b^2 c^2 d^3 + 35 a^4 b c d^4 + 3 a^5 d^5) x^6 + \\
 & a c (b^6 c^6 + 21 a b^5 c^5 d + 105 a^2 b^4 c^4 d^2 + 175 a^3 b^3 c^3 d^3 + 105 a^4 b^2 c^2 d^4 + 21 a^5 b c d^5 + a^6 d^6) x^7 + \\
 & \frac{1}{8} (b^7 c^7 + 49 a b^6 c^6 d + 441 a^2 b^5 c^5 d^2 + 1225 a^3 b^4 c^4 d^3 + \\
 & \quad 1225 a^4 b^3 c^3 d^4 + 441 a^5 b^2 c^2 d^5 + 49 a^6 b c d^6 + a^7 d^7) x^8 + \\
 & \frac{7}{9} b d (b^6 c^6 + 21 a b^5 c^5 d + 105 a^2 b^4 c^4 d^2 + 175 a^3 b^3 c^3 d^3 + 105 a^4 b^2 c^2 d^4 + 21 a^5 b c d^5 + a^6 d^6) x^9 + \\
 & \frac{7}{10} b^2 d^2 (3 b^5 c^5 + 35 a b^4 c^4 d + 105 a^2 b^3 c^3 d^2 + 105 a^3 b^2 c^2 d^3 + 35 a^4 b c d^4 + 3 a^5 d^5) x^{10} + \\
 & \frac{7}{11} b^3 d^3 (5 b^4 c^4 + 35 a b^3 c^3 d + 63 a^2 b^2 c^2 d^2 + 35 a^3 b c d^3 + 5 a^4 d^4) x^{11} + \\
 & \frac{7}{12} b^4 d^4 (5 b^3 c^3 + 21 a b^2 c^2 d + 21 a^2 b c d^2 + 5 a^3 d^3) x^{12} + \\
 & \frac{7}{13} b^5 d^5 (3 b^2 c^2 + 7 a b c d + 3 a^2 d^2) x^{13} + \frac{1}{2} b^6 d^6 (b c + a d) x^{14} + \frac{1}{15} b^7 d^7 x^{15}
 \end{aligned}$$

**Problem 1276: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^6 (c + d x)^7 dx$$

Optimal (type 1, 173 leaves, 2 steps):

$$\begin{aligned}
 & \frac{(b c - a d)^6 (c + d x)^8}{8 d^7} - \frac{2 b (b c - a d)^5 (c + d x)^9}{3 d^7} + \frac{3 b^2 (b c - a d)^4 (c + d x)^{10}}{2 d^7} - \\
 & \frac{20 b^3 (b c - a d)^3 (c + d x)^{11}}{11 d^7} + \frac{5 b^4 (b c - a d)^2 (c + d x)^{12}}{4 d^7} - \frac{6 b^5 (b c - a d) (c + d x)^{13}}{13 d^7} + \frac{b^6 (c + d x)^{14}}{14 d^7}
 \end{aligned}$$

Result (type 1, 684 leaves):

$$\begin{aligned}
 & a^6 c^7 x + \frac{1}{2} a^5 c^6 (6 b c + 7 a d) x^2 + a^4 c^5 (5 b^2 c^2 + 14 a b c d + 7 a^2 d^2) x^3 + \\
 & \frac{1}{4} a^3 c^4 (20 b^3 c^3 + 105 a b^2 c^2 d + 126 a^2 b c d^2 + 35 a^3 d^3) x^4 + \\
 & a^2 c^3 (3 b^4 c^4 + 28 a b^3 c^3 d + 63 a^2 b^2 c^2 d^2 + 42 a^3 b c d^3 + 7 a^4 d^4) x^5 + \\
 & \frac{1}{2} a c^2 (2 b^5 c^5 + 35 a b^4 c^4 d + 140 a^2 b^3 c^3 d^2 + 175 a^3 b^2 c^2 d^3 + 70 a^4 b c d^4 + 7 a^5 d^5) x^6 + \\
 & \frac{1}{7} c (b^6 c^6 + 42 a b^5 c^5 d + 315 a^2 b^4 c^4 d^2 + 700 a^3 b^3 c^3 d^3 + 525 a^4 b^2 c^2 d^4 + 126 a^5 b c d^5 + 7 a^6 d^6) x^7 + \\
 & \frac{1}{8} d (7 b^6 c^6 + 126 a b^5 c^5 d + 525 a^2 b^4 c^4 d^2 + 700 a^3 b^3 c^3 d^3 + 315 a^4 b^2 c^2 d^4 + 42 a^5 b c d^5 + a^6 d^6) x^8 + \\
 & \frac{1}{3} b d^2 (7 b^5 c^5 + 70 a b^4 c^4 d + 175 a^2 b^3 c^3 d^2 + 140 a^3 b^2 c^2 d^3 + 35 a^4 b c d^4 + 2 a^5 d^5) x^9 + \\
 & \frac{1}{2} b^2 d^3 (7 b^4 c^4 + 42 a b^3 c^3 d + 63 a^2 b^2 c^2 d^2 + 28 a^3 b c d^3 + 3 a^4 d^4) x^{10} + \\
 & \frac{1}{11} b^3 d^4 (35 b^3 c^3 + 126 a b^2 c^2 d + 105 a^2 b c d^2 + 20 a^3 d^3) x^{11} + \\
 & \frac{1}{4} b^4 d^5 (7 b^2 c^2 + 14 a b c d + 5 a^2 d^2) x^{12} + \frac{1}{13} b^5 d^6 (7 b c + 6 a d) x^{13} + \frac{1}{14} b^6 d^7 x^{14}
 \end{aligned}$$

**Problem 1277: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^5 (c + d x)^7 dx$$

Optimal (type 1, 144 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{(b c - a d)^5 (c + d x)^8}{8 d^6} + \frac{5 b (b c - a d)^4 (c + d x)^9}{9 d^6} - \frac{b^2 (b c - a d)^3 (c + d x)^{10}}{d^6} + \\
 & \frac{10 b^3 (b c - a d)^2 (c + d x)^{11}}{11 d^6} - \frac{5 b^4 (b c - a d) (c + d x)^{12}}{12 d^6} + \frac{b^5 (c + d x)^{13}}{13 d^6}
 \end{aligned}$$

Result (type 1, 574 leaves):

$$\begin{aligned}
& a^5 c^7 x + \frac{1}{2} a^4 c^6 (5 b c + 7 a d) x^2 + \frac{1}{3} a^3 c^5 (10 b^2 c^2 + 35 a b c d + 21 a^2 d^2) x^3 + \\
& \frac{5}{4} a^2 c^4 (2 b^3 c^3 + 14 a b^2 c^2 d + 21 a^2 b c d^2 + 7 a^3 d^3) x^4 + \\
& a c^3 (b^4 c^4 + 14 a b^3 c^3 d + 42 a^2 b^2 c^2 d^2 + 35 a^3 b c d^3 + 7 a^4 d^4) x^5 + \\
& \frac{1}{6} c^2 (b^5 c^5 + 35 a b^4 c^4 d + 210 a^2 b^3 c^3 d^2 + 350 a^3 b^2 c^2 d^3 + 175 a^4 b c d^4 + 21 a^5 d^5) x^6 + \\
& c d (b^5 c^5 + 15 a b^4 c^4 d + 50 a^2 b^3 c^3 d^2 + 50 a^3 b^2 c^2 d^3 + 15 a^4 b c d^4 + a^5 d^5) x^7 + \\
& \frac{1}{8} d^2 (21 b^5 c^5 + 175 a b^4 c^4 d + 350 a^2 b^3 c^3 d^2 + 210 a^3 b^2 c^2 d^3 + 35 a^4 b c d^4 + a^5 d^5) x^8 + \\
& \frac{5}{9} b d^3 (7 b^4 c^4 + 35 a b^3 c^3 d + 42 a^2 b^2 c^2 d^2 + 14 a^3 b c d^3 + a^4 d^4) x^9 + \\
& \frac{1}{2} b^2 d^4 (7 b^3 c^3 + 21 a b^2 c^2 d + 14 a^2 b c d^2 + 2 a^3 d^3) x^{10} + \\
& \frac{1}{11} b^3 d^5 (21 b^2 c^2 + 35 a b c d + 10 a^2 d^2) x^{11} + \frac{1}{12} b^4 d^6 (7 b c + 5 a d) x^{12} + \frac{1}{13} b^5 d^7 x^{13}
\end{aligned}$$

### Problem 1278: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^4 (c + d x)^7 dx$$

Optimal (type 1, 119 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b c - a d)^4 (c + d x)^8}{8 d^5} - \frac{4 b (b c - a d)^3 (c + d x)^9}{9 d^5} + \\
& \frac{3 b^2 (b c - a d)^2 (c + d x)^{10}}{5 d^5} - \frac{4 b^3 (b c - a d) (c + d x)^{11}}{11 d^5} + \frac{b^4 (c + d x)^{12}}{12 d^5}
\end{aligned}$$

Result (type 1, 473 leaves):

$$\begin{aligned}
& a^4 c^7 x + \frac{1}{2} a^3 c^6 (4 b c + 7 a d) x^2 + \frac{1}{3} a^2 c^5 (6 b^2 c^2 + 28 a b c d + 21 a^2 d^2) x^3 + \\
& \frac{1}{4} a c^4 (4 b^3 c^3 + 42 a b^2 c^2 d + 84 a^2 b c d^2 + 35 a^3 d^3) x^4 + \\
& \frac{1}{5} c^3 (b^4 c^4 + 28 a b^3 c^3 d + 126 a^2 b^2 c^2 d^2 + 140 a^3 b c d^3 + 35 a^4 d^4) x^5 + \\
& \frac{7}{6} c^2 d (b^4 c^4 + 12 a b^3 c^3 d + 30 a^2 b^2 c^2 d^2 + 20 a^3 b c d^3 + 3 a^4 d^4) x^6 + \\
& c d^2 (3 b^4 c^4 + 20 a b^3 c^3 d + 30 a^2 b^2 c^2 d^2 + 12 a^3 b c d^3 + a^4 d^4) x^7 + \\
& \frac{1}{8} d^3 (35 b^4 c^4 + 140 a b^3 c^3 d + 126 a^2 b^2 c^2 d^2 + 28 a^3 b c d^3 + a^4 d^4) x^8 + \\
& \frac{1}{9} b d^4 (35 b^3 c^3 + 84 a b^2 c^2 d + 42 a^2 b c d^2 + 4 a^3 d^3) x^9 + \\
& \frac{1}{10} b^2 d^5 (21 b^2 c^2 + 28 a b c d + 6 a^2 d^2) x^{10} + \frac{1}{11} b^3 d^6 (7 b c + 4 a d) x^{11} + \frac{1}{12} b^4 d^7 x^{12}
\end{aligned}$$

### Problem 1279: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^3 (c + d x)^7 dx$$

Optimal (type 1, 92 leaves, 2 steps):

$$-\frac{(bc - ad)^3 (c + dx)^8}{8d^4} + \frac{b(bc - ad)^2 (c + dx)^9}{3d^4} - \frac{3b^2(bc - ad)(c + dx)^{10}}{10d^4} + \frac{b^3(c + dx)^{11}}{11d^4}$$

Result (type 1, 360 leaves):

$$\begin{aligned} & a^3 c^7 x + \frac{1}{2} a^2 c^6 (3bc + 7ad) x^2 + a c^5 (b^2 c^2 + 7abcd + 7a^2 d^2) x^3 + \\ & \frac{1}{4} c^4 (b^3 c^3 + 21a b^2 c^2 d + 63a^2 b c d^2 + 35a^3 d^3) x^4 + \\ & \frac{7}{5} c^3 d (b^3 c^3 + 9a b^2 c^2 d + 15a^2 b c d^2 + 5a^3 d^3) x^5 + \frac{7}{2} c^2 d^2 (b^3 c^3 + 5a b^2 c^2 d + 5a^2 b c d^2 + a^3 d^3) x^6 + \\ & c d^3 (5b^3 c^3 + 15a b^2 c^2 d + 9a^2 b c d^2 + a^3 d^3) x^7 + \frac{1}{8} d^4 (35b^3 c^3 + 63a b^2 c^2 d + 21a^2 b c d^2 + a^3 d^3) x^8 + \\ & \frac{1}{3} b d^5 (7b^2 c^2 + 7abcd + a^2 d^2) x^9 + \frac{1}{10} b^2 d^6 (7bc + 3ad) x^{10} + \frac{1}{11} b^3 d^7 x^{11} \end{aligned}$$

### Problem 1280: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^2 (c + d x)^7 dx$$

Optimal (type 1, 65 leaves, 2 steps):

$$\frac{(bc - ad)^2 (c + dx)^8}{8d^3} - \frac{2b(bc - ad)(c + dx)^9}{9d^3} + \frac{b^2(c + dx)^{10}}{10d^3}$$

Result (type 1, 261 leaves):

$$\begin{aligned} & a^2 c^7 x + \frac{1}{2} a c^6 (2bc + 7ad) x^2 + \frac{1}{3} c^5 (b^2 c^2 + 14abcd + 21a^2 d^2) x^3 + \\ & \frac{7}{4} c^4 d (b^2 c^2 + 6abcd + 5a^2 d^2) x^4 + \frac{7}{5} c^3 d^2 (3b^2 c^2 + 10abcd + 5a^2 d^2) x^5 + \\ & \frac{7}{6} c^2 d^3 (5b^2 c^2 + 10abcd + 3a^2 d^2) x^6 + c d^4 (5b^2 c^2 + 6abcd + a^2 d^2) x^7 + \\ & \frac{1}{8} d^5 (21b^2 c^2 + 14abcd + a^2 d^2) x^8 + \frac{1}{9} b d^6 (7bc + 2ad) x^9 + \frac{1}{10} b^2 d^7 x^{10} \end{aligned}$$

### Problem 1281: Result more than twice size of optimal antiderivative.

$$\int (a + b x) (c + d x)^7 dx$$

Optimal (type 1, 38 leaves, 2 steps):

$$-\frac{(bc-ad)(c+dx)^8}{8d^2} + \frac{b(c+dx)^9}{9d^2}$$

Result (type 1, 151 leaves):

$$ac^7x + \frac{1}{2}c^6(bc+7ad)x^2 + \frac{7}{3}c^5d(bc+3ad)x^3 + \frac{7}{4}c^4d^2(3bc+5ad)x^4 + 7c^3d^3(bc+ad)x^5 + \frac{7}{6}c^2d^4(5bc+3ad)x^6 + cd^5(3bc+ad)x^7 + \frac{1}{8}d^6(7bc+ad)x^8 + \frac{1}{9}bd^7x^9$$

**Problem 1284: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+dx)^7}{(a+bx)^2} dx$$

Optimal (type 3, 187 leaves, 2 steps):

$$\frac{21d^2(bc-ad)^5x}{b^7} - \frac{(bc-ad)^7}{b^8(a+bx)} + \frac{35d^3(bc-ad)^4(a+bx)^2}{2b^8} + \frac{35d^4(bc-ad)^3(a+bx)^3}{3b^8} + \frac{21d^5(bc-ad)^2(a+bx)^4}{4b^8} + \frac{7d^6(bc-ad)(a+bx)^5}{5b^8} + \frac{d^7(a+bx)^6}{6b^8} + \frac{7d(bc-ad)^6 \text{Log}[a+bx]}{b^8}$$

Result (type 3, 388 leaves):

$$\frac{1}{60b^8(a+bx)} (60a^7d^7 - 60a^6bd^6(7c+6dx) + 210a^5b^2d^5(6c^2+10cdx-d^2x^2) + 70a^4b^3d^4(-30c^3-72c^2dx+18cd^2x^2+d^3x^3) - 35a^3b^4d^3(-60c^4-180c^3dx+90c^2d^2x^2+12cd^3x^3+d^4x^4) + 21a^2b^5d^2(-60c^5-200c^4dx+200c^3d^2x^2+50c^2d^3x^3+10cd^4x^4+d^5x^5) - 7ab^6d(-60c^6-180c^5dx+450c^4d^2x^2+200c^3d^3x^3+75c^2d^4x^4+18cd^5x^5+2d^6x^6) + b^7(-60c^7+1260c^5d^2x^2+1050c^4d^3x^3+700c^3d^4x^4+315c^2d^5x^5+84cd^6x^6+10d^7x^7) + 420d(bc-ad)^6(a+bx) \text{Log}[a+bx])$$

**Problem 1285: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+dx)^7}{(a+bx)^3} dx$$

Optimal (type 3, 185 leaves, 2 steps):

$$\frac{35d^3(bc-ad)^4x}{b^7} - \frac{(bc-ad)^7}{2b^8(a+bx)^2} - \frac{7d(bc-ad)^6}{b^8(a+bx)} + \frac{35d^4(bc-ad)^3(a+bx)^2}{2b^8} + \frac{7d^5(bc-ad)^2(a+bx)^3}{b^8} + \frac{7d^6(bc-ad)(a+bx)^4}{4b^8} + \frac{d^7(a+bx)^5}{5b^8} + \frac{21d^2(bc-ad)^5 \text{Log}[a+bx]}{b^8}$$

Result (type 3, 389 leaves):



$$\frac{1}{20 b^8 (a + b x)^2} \left( -130 a^7 d^7 + 10 a^6 b d^6 (77 c + 16 d x) + \right. \\
 10 a^5 b^2 d^5 (-189 c^2 - 56 c d x + 50 d^2 x^2) + 70 a^4 b^3 d^4 (35 c^3 + 6 c^2 d x - 34 c d^2 x^2 + 2 d^3 x^3) - \\
 35 a^3 b^4 d^3 (50 c^4 - 20 c^3 d x - 126 c^2 d^2 x^2 + 20 c d^3 x^3 + d^4 x^4) + \\
 7 a^2 b^5 d^2 (90 c^5 - 200 c^4 d x - 550 c^3 d^2 x^2 + 200 c^2 d^3 x^3 + 25 c d^4 x^4 + 2 d^5 x^5) - \\
 7 a b^6 d (10 c^6 - 120 c^5 d x - 200 c^4 d^2 x^2 + 200 c^3 d^3 x^3 + 50 c^2 d^4 x^4 + 10 c d^5 x^5 + d^6 x^6) + \\
 \left. b^7 (-10 c^7 - 140 c^6 d x + 700 c^4 d^3 x^3 + 350 c^3 d^4 x^4 + 140 c^2 d^5 x^5 + 35 c d^6 x^6 + 4 d^7 x^7) - \right. \\
 \left. 420 d^2 (-b c + a d)^5 (a + b x)^2 \operatorname{Log}[a + b x] \right)$$

**Problem 1288: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^7}{(a + b x)^6} dx$$

Optimal (type 3, 181 leaves, 2 steps):

$$\frac{d^6 (7 b c - 6 a d) x}{b^7} + \frac{d^7 x^2}{2 b^6} - \frac{(b c - a d)^7}{5 b^8 (a + b x)^5} - \frac{7 d (b c - a d)^6}{4 b^8 (a + b x)^4} - \\
 \frac{7 d^2 (b c - a d)^5}{b^8 (a + b x)^3} - \frac{35 d^3 (b c - a d)^4}{2 b^8 (a + b x)^2} - \frac{35 d^4 (b c - a d)^3}{b^8 (a + b x)} + \frac{21 d^5 (b c - a d)^2 \operatorname{Log}[a + b x]}{b^8}$$

Result (type 3, 389 leaves):

$$\frac{1}{20 b^8 (a + b x)^5} \left( 459 a^7 d^7 + 3 a^6 b d^6 (-406 c + 625 d x) + \right. \\
 a^5 b^2 d^5 (959 c^2 - 5250 c d x + 2700 d^2 x^2) + 5 a^4 b^3 d^4 (-28 c^3 + 875 c^2 d x - 1680 c d^2 x^2 + 260 d^3 x^3) - \\
 5 a^3 b^4 d^3 (7 c^4 + 140 c^3 d x - 1540 c^2 d^2 x^2 + 1120 c d^3 x^3 + 80 d^4 x^4) - \\
 a^2 b^5 d^2 (14 c^5 + 175 c^4 d x + 1400 c^3 d^2 x^2 - 6300 c^2 d^3 x^3 + 700 c d^4 x^4 + 500 d^5 x^5) - \\
 7 a b^6 d (c^6 + 10 c^5 d x + 50 c^4 d^2 x^2 + 200 c^3 d^3 x^3 - 300 c^2 d^4 x^4 - 100 c d^5 x^5 + 10 d^6 x^6) - \\
 \left. b^7 (4 c^7 + 35 c^6 d x + 140 c^5 d^2 x^2 + 350 c^4 d^3 x^3 + 700 c^3 d^4 x^4 - 140 c d^6 x^6 - 10 d^7 x^7) + \right. \\
 \left. 420 d^5 (b c - a d)^2 (a + b x)^5 \operatorname{Log}[a + b x] \right)$$

**Problem 1289: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^7}{(a + b x)^7} dx$$

Optimal (type 3, 186 leaves, 2 steps):

$$\frac{d^7 x}{b^7} - \frac{(b c - a d)^7}{6 b^8 (a + b x)^6} - \frac{7 d (b c - a d)^6}{5 b^8 (a + b x)^5} - \frac{21 d^2 (b c - a d)^5}{4 b^8 (a + b x)^4} - \\
 \frac{35 d^3 (b c - a d)^4}{3 b^8 (a + b x)^3} - \frac{35 d^4 (b c - a d)^3}{2 b^8 (a + b x)^2} - \frac{21 d^5 (b c - a d)^2}{b^8 (a + b x)} + \frac{7 d^6 (b c - a d) \operatorname{Log}[a + b x]}{b^8}$$

Result (type 3, 390 leaves):

$$\begin{aligned}
& - \frac{1}{60 b^8 (a + b x)^6} \left( 669 a^7 d^7 + 3 a^6 b d^6 (-343 c + 1198 d x) + \right. \\
& \quad 3 a^5 b^2 d^5 (70 c^2 - 1918 c d x + 2575 d^2 x^2) + 5 a^4 b^3 d^4 (14 c^3 + 252 c^2 d x - 2625 c d^2 x^2 + 1640 d^3 x^3) + \\
& \quad 5 a^3 b^4 d^3 (7 c^4 + 84 c^3 d x + 630 c^2 d^2 x^2 - 3080 c d^3 x^3 + 810 d^4 x^4) + \\
& \quad 3 a^2 b^5 d^2 (7 c^5 + 70 c^4 d x + 350 c^3 d^2 x^2 + 1400 c^2 d^3 x^3 - 3150 c d^4 x^4 + 120 d^5 x^5) + \\
& \quad a b^6 d (14 c^6 + 126 c^5 d x + 525 c^4 d^2 x^2 + 1400 c^3 d^3 x^3 + 3150 c^2 d^4 x^4 - 2520 c d^5 x^5 - 360 d^6 x^6) + \\
& \quad \left. b^7 (10 c^7 + 84 c^6 d x + 315 c^5 d^2 x^2 + 700 c^4 d^3 x^3 + 1050 c^3 d^4 x^4 + 1260 c^2 d^5 x^5 - 60 d^7 x^7) + \right. \\
& \quad \left. 420 d^6 (-b c + a d) (a + b x)^6 \operatorname{Log}[a + b x] \right)
\end{aligned}$$

**Problem 1291: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^7}{(a + b x)^9} dx$$

Optimal (type 1, 28 leaves, 1 step):

$$- \frac{(c + d x)^8}{8 (b c - a d) (a + b x)^8}$$

Result (type 1, 353 leaves):

$$\begin{aligned}
& - \frac{1}{8 b^8 (a + b x)^8} \left( a^7 d^7 + a^6 b d^6 (c + 8 d x) + a^5 b^2 d^5 (c^2 + 8 c d x + 28 d^2 x^2) + a^4 b^3 d^4 \right. \\
& \quad (c^3 + 8 c^2 d x + 28 c d^2 x^2 + 56 d^3 x^3) + a^3 b^4 d^3 (c^4 + 8 c^3 d x + 28 c^2 d^2 x^2 + 56 c d^3 x^3 + 70 d^4 x^4) + \\
& \quad a^2 b^5 d^2 (c^5 + 8 c^4 d x + 28 c^3 d^2 x^2 + 56 c^2 d^3 x^3 + 70 c d^4 x^4 + 56 d^5 x^5) + \\
& \quad a b^6 d (c^6 + 8 c^5 d x + 28 c^4 d^2 x^2 + 56 c^3 d^3 x^3 + 70 c^2 d^4 x^4 + 56 c d^5 x^5 + 28 d^6 x^6) + \\
& \quad \left. b^7 (c^7 + 8 c^6 d x + 28 c^5 d^2 x^2 + 56 c^4 d^3 x^3 + 70 c^3 d^4 x^4 + 56 c^2 d^5 x^5 + 28 c d^6 x^6 + 8 d^7 x^7) \right)
\end{aligned}$$

**Problem 1292: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^7}{(a + b x)^{10}} dx$$

Optimal (type 1, 58 leaves, 2 steps):

$$- \frac{(c + d x)^8}{9 (b c - a d) (a + b x)^9} + \frac{d (c + d x)^8}{72 (b c - a d)^2 (a + b x)^8}$$

Result (type 1, 367 leaves):

$$\begin{aligned}
& - \frac{1}{72 b^8 (a + b x)^9} \left( a^7 d^7 + a^6 b d^6 (2 c + 9 d x) + \right. \\
& \quad 3 a^5 b^2 d^5 (c^2 + 6 c d x + 12 d^2 x^2) + a^4 b^3 d^4 (4 c^3 + 27 c^2 d x + 72 c d^2 x^2 + 84 d^3 x^3) + \\
& \quad a^3 b^4 d^3 (5 c^4 + 36 c^3 d x + 108 c^2 d^2 x^2 + 168 c d^3 x^3 + 126 d^4 x^4) + \\
& \quad 3 a^2 b^5 d^2 (2 c^5 + 15 c^4 d x + 48 c^3 d^2 x^2 + 84 c^2 d^3 x^3 + 84 c d^4 x^4 + 42 d^5 x^5) + \\
& \quad a b^6 d (7 c^6 + 54 c^5 d x + 180 c^4 d^2 x^2 + 336 c^3 d^3 x^3 + 378 c^2 d^4 x^4 + 252 c d^5 x^5 + 84 d^6 x^6) + \\
& \quad \left. b^7 (8 c^7 + 63 c^6 d x + 216 c^5 d^2 x^2 + 420 c^4 d^3 x^3 + 504 c^3 d^4 x^4 + 378 c^2 d^5 x^5 + 168 c d^6 x^6 + 36 d^7 x^7) \right)
\end{aligned}$$

### Problem 1293: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+dx)^7}{(a+bx)^{11}} dx$$

Optimal (type 1, 89 leaves, 3 steps):

$$-\frac{(c+dx)^8}{10(bc-ad)(a+bx)^{10}} + \frac{d(c+dx)^8}{45(bc-ad)^2(a+bx)^9} - \frac{d^2(c+dx)^8}{360(bc-ad)^3(a+bx)^8}$$

Result (type 1, 371 leaves):

$$-\frac{1}{360b^8(a+bx)^{10}} (a^7d^7 + a^6bd^6(3c+10dx) + 3a^5b^2d^5(2c^2+10cdx+15d^2x^2) + 5a^4b^3d^4(2c^3+12c^2dx+27cd^2x^2+24d^3x^3) + 5a^3b^4d^3(3c^4+20c^3dx+54c^2d^2x^2+72cd^3x^3+42d^4x^4) + 3a^2b^5d^2(7c^5+50c^4dx+150c^3d^2x^2+240c^2d^3x^3+210cd^4x^4+84d^5x^5) + ab^6d(28c^6+210c^5dx+675c^4d^2x^2+1200c^3d^3x^3+1260c^2d^4x^4+756cd^5x^5+210d^6x^6) + b^7(36c^7+280c^6dx+945c^5d^2x^2+1800c^4d^3x^3+2100c^3d^4x^4+1512c^2d^5x^5+630cd^6x^6+120d^7x^7))$$

### Problem 1294: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+dx)^7}{(a+bx)^{12}} dx$$

Optimal (type 1, 120 leaves, 4 steps):

$$-\frac{(c+dx)^8}{11(bc-ad)(a+bx)^{11}} + \frac{3d(c+dx)^8}{110(bc-ad)^2(a+bx)^{10}} - \frac{d^2(c+dx)^8}{165(bc-ad)^3(a+bx)^9} + \frac{d^3(c+dx)^8}{1320(bc-ad)^4(a+bx)^8}$$

Result (type 1, 369 leaves):

$$-\frac{1}{1320b^8(a+bx)^{11}} (a^7d^7 + a^6bd^6(4c+11dx) + a^5b^2d^5(10c^2+44cdx+55d^2x^2) + 5a^4b^3d^4(4c^3+22c^2dx+44cd^2x^2+33d^3x^3) + 5a^3b^4d^3(7c^4+44c^3dx+110c^2d^2x^2+132cd^3x^3+66d^4x^4) + a^2b^5d^2(56c^5+385c^4dx+1100c^3d^2x^2+1650c^2d^3x^3+1320cd^4x^4+462d^5x^5) + ab^6d(84c^6+616c^5dx+1925c^4d^2x^2+3300c^3d^3x^3+3300c^2d^4x^4+1848cd^5x^5+462d^6x^6) + b^7(120c^7+924c^6dx+3080c^5d^2x^2+5775c^4d^3x^3+6600c^3d^4x^4+4620c^2d^5x^5+1848cd^6x^6+330d^7x^7))$$

### Problem 1295: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+dx)^7}{(a+bx)^{13}} dx$$

Optimal (type 1, 151 leaves, 5 steps):

$$-\frac{(c+dx)^8}{12(bc-ad)(a+bx)^{12}} + \frac{d(c+dx)^8}{33(bc-ad)^2(a+bx)^{11}} - \frac{d^2(c+dx)^8}{110(bc-ad)^3(a+bx)^{10}} + \frac{d^3(c+dx)^8}{495(bc-ad)^4(a+bx)^9} - \frac{d^4(c+dx)^8}{3960(bc-ad)^5(a+bx)^8}$$

Result (type 1, 371 leaves):

$$-\frac{1}{3960b^8(a+bx)^{12}} (a^7d^7 + a^6bd^6(5c+12dx) + 3a^5b^2d^5(5c^2+20cdx+22d^2x^2) + 5a^4b^3d^4(7c^3+36c^2dx+66cd^2x^2+44d^3x^3) + 5a^3b^4d^3(14c^4+84c^3dx+198c^2d^2x^2+220cd^3x^3+99d^4x^4) + 3a^2b^5d^2(42c^5+280c^4dx+770c^3d^2x^2+1100c^2d^3x^3+825cd^4x^4+264d^5x^5) + ab^6d(210c^6+1512c^5dx+4620c^4d^2x^2+7700c^3d^3x^3+7425c^2d^4x^4+3960cd^5x^5+924d^6x^6) + b^7(330c^7+2520c^6dx+8316c^5d^2x^2+15400c^4d^3x^3+17325c^3d^4x^4+11880c^2d^5x^5+4620cd^6x^6+792d^7x^7))$$

### Problem 1299: Result more than twice size of optimal antiderivative.

$$\int (a+bx)^{12} (c+dx)^{10} dx$$

Optimal (type 1, 275 leaves, 2 steps):

$$\frac{(bc-ad)^{10}(a+bx)^{13}}{13b^{11}} + \frac{5d(bc-ad)^9(a+bx)^{14}}{7b^{11}} + \frac{3d^2(bc-ad)^8(a+bx)^{15}}{b^{11}} + \frac{15d^3(bc-ad)^7(a+bx)^{16}}{2b^{11}} + \frac{210d^4(bc-ad)^6(a+bx)^{17}}{17b^{11}} + \frac{14d^5(bc-ad)^5(a+bx)^{18}}{b^{11}} + \frac{210d^6(bc-ad)^4(a+bx)^{19}}{19b^{11}} + \frac{6d^7(bc-ad)^3(a+bx)^{20}}{b^{11}} + \frac{15d^8(bc-ad)^2(a+bx)^{21}}{7b^{11}} + \frac{5d^9(bc-ad)(a+bx)^{22}}{11b^{11}} + \frac{d^{10}(a+bx)^{23}}{23b^{11}}$$

Result (type 1, 1817 leaves):

$$\begin{aligned}
 & a^{12} c^{10} x + a^{11} c^9 (6 b c + 5 a d) x^2 + a^{10} c^8 (22 b^2 c^2 + 40 a b c d + 15 a^2 d^2) x^3 + \\
 & 5 a^9 c^7 (11 b^3 c^3 + 33 a b^2 c^2 d + 27 a^2 b c d^2 + 6 a^3 d^3) x^4 + \\
 & a^8 c^6 (99 b^4 c^4 + 440 a b^3 c^3 d + 594 a^2 b^2 c^2 d^2 + 288 a^3 b c d^3 + 42 a^4 d^4) x^5 + \\
 & 3 a^7 c^5 (44 b^5 c^5 + 275 a b^4 c^4 d + 550 a^2 b^3 c^3 d^2 + 440 a^3 b^2 c^2 d^3 + 140 a^4 b c d^4 + 14 a^5 d^5) x^6 + \\
 & \frac{3}{7} a^6 c^4 (308 b^6 c^6 + 2640 a b^5 c^5 d + 7425 a^2 b^4 c^4 d^2 + 8800 a^3 b^3 c^3 d^3 + 4620 a^4 b^2 c^2 d^4 + \\
 & 1008 a^5 b c d^5 + 70 a^6 d^6) x^7 + 3 a^5 c^3 (33 b^7 c^7 + 385 a b^6 c^6 d + 1485 a^2 b^5 c^5 d^2 + \\
 & 2475 a^3 b^4 c^4 d^3 + 1925 a^4 b^3 c^3 d^4 + 693 a^5 b^2 c^2 d^5 + 105 a^6 b c d^6 + 5 a^7 d^7) x^8 + \\
 & 5 a^4 c^2 (11 b^8 c^8 + 176 a b^7 c^7 d + 924 a^2 b^6 c^6 d^2 + 2112 a^3 b^5 c^5 d^3 + 2310 a^4 b^4 c^4 d^4 + \\
 & 1232 a^5 b^3 c^3 d^5 + 308 a^6 b^2 c^2 d^6 + 32 a^7 b c d^7 + a^8 d^8) x^9 + \\
 & a^3 c (22 b^9 c^9 + 495 a b^8 c^8 d + 3564 a^2 b^7 c^7 d^2 + 11088 a^3 b^6 c^6 d^3 + 16632 a^4 b^5 c^5 d^4 + \\
 & 12474 a^5 b^4 c^4 d^5 + 4620 a^6 b^3 c^3 d^6 + 792 a^7 b^2 c^2 d^7 + 54 a^8 b c d^8 + a^9 d^9) x^{10} + \\
 & \frac{1}{11} a^2 (66 b^{10} c^{10} + 2200 a b^9 c^9 d + 22275 a^2 b^8 c^8 d^2 + 95040 a^3 b^7 c^7 d^3 + 194040 a^4 b^6 c^6 d^4 + \\
 & 199584 a^5 b^5 c^5 d^5 + 103950 a^6 b^4 c^4 d^6 + 26400 a^7 b^3 c^3 d^7 + 2970 a^8 b^2 c^2 d^8 + 120 a^9 b c d^9 + a^{10} d^{10}) \\
 & x^{11} + a b (b^{10} c^{10} + 55 a b^9 c^9 d + 825 a^2 b^8 c^8 d^2 + 4950 a^3 b^7 c^7 d^3 + 13860 a^4 b^6 c^6 d^4 + 19404 a^5 b^5 c^5 d^5 + \\
 & 13860 a^6 b^4 c^4 d^6 + 4950 a^7 b^3 c^3 d^7 + 825 a^8 b^2 c^2 d^8 + 55 a^9 b c d^9 + a^{10} d^{10}) x^{12} + \frac{1}{13} b^2 \\
 & (b^{10} c^{10} + 120 a b^9 c^9 d + 2970 a^2 b^8 c^8 d^2 + 26400 a^3 b^7 c^7 d^3 + 103950 a^4 b^6 c^6 d^4 + 199584 a^5 b^5 c^5 d^5 + \\
 & 194040 a^6 b^4 c^4 d^6 + 95040 a^7 b^3 c^3 d^7 + 22275 a^8 b^2 c^2 d^8 + 2200 a^9 b c d^9 + 66 a^{10} d^{10}) x^{13} + \\
 & \frac{5}{7} b^3 d (b^9 c^9 + 54 a b^8 c^8 d + 792 a^2 b^7 c^7 d^2 + 4620 a^3 b^6 c^6 d^3 + 12474 a^4 b^5 c^5 d^4 + \\
 & 16632 a^5 b^4 c^4 d^5 + 11088 a^6 b^3 c^3 d^6 + 3564 a^7 b^2 c^2 d^7 + 495 a^8 b c d^8 + 22 a^9 d^9) x^{14} + \\
 & 3 b^4 d^2 (b^8 c^8 + 32 a b^7 c^7 d + 308 a^2 b^6 c^6 d^2 + 1232 a^3 b^5 c^5 d^3 + 2310 a^4 b^4 c^4 d^4 + \\
 & 2112 a^5 b^3 c^3 d^5 + 924 a^6 b^2 c^2 d^6 + 176 a^7 b c d^7 + 11 a^8 d^8) x^{15} + \\
 & \frac{3}{2} b^5 d^3 (5 b^7 c^7 + 105 a b^6 c^6 d + 693 a^2 b^5 c^5 d^2 + 1925 a^3 b^4 c^4 d^3 + 2475 a^4 b^3 c^3 d^4 + \\
 & 1485 a^5 b^2 c^2 d^5 + 385 a^6 b c d^6 + 33 a^7 d^7) x^{16} + \frac{3}{17} b^6 d^4 (70 b^6 c^6 + 1008 a b^5 c^5 d + \\
 & 4620 a^2 b^4 c^4 d^2 + 8800 a^3 b^3 c^3 d^3 + 7425 a^4 b^2 c^2 d^4 + 2640 a^5 b c d^5 + 308 a^6 d^6) x^{17} + \\
 & b^7 d^5 (14 b^5 c^5 + 140 a b^4 c^4 d + 440 a^2 b^3 c^3 d^2 + 550 a^3 b^2 c^2 d^3 + 275 a^4 b c d^4 + 44 a^5 d^5) x^{18} + \\
 & \frac{5}{19} b^8 d^6 (42 b^4 c^4 + 288 a b^3 c^3 d + 594 a^2 b^2 c^2 d^2 + 440 a^3 b c d^3 + 99 a^4 d^4) x^{19} + \\
 & b^9 d^7 (6 b^3 c^3 + 27 a b^2 c^2 d + 33 a^2 b c d^2 + 11 a^3 d^3) x^{20} + \\
 & \frac{1}{7} b^{10} d^8 (15 b^2 c^2 + 40 a b c d + 22 a^2 d^2) x^{21} + \\
 & \frac{1}{11} b^{11} d^9 (5 b c + 6 a d) x^{22} + \frac{1}{23} b^{12} d^{10} x^{23}
 \end{aligned}$$

**Problem 1300: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{11} (c + d x)^{10} dx$$

Optimal (type 1, 279 leaves, 2 steps):

$$\begin{aligned}
& \frac{(bc-ad)^{10} (a+bx)^{12}}{12 b^{11}} + \frac{10 d (bc-ad)^9 (a+bx)^{13}}{13 b^{11}} + \\
& \frac{45 d^2 (bc-ad)^8 (a+bx)^{14}}{14 b^{11}} + \frac{8 d^3 (bc-ad)^7 (a+bx)^{15}}{b^{11}} + \frac{105 d^4 (bc-ad)^6 (a+bx)^{16}}{8 b^{11}} + \\
& \frac{252 d^5 (bc-ad)^5 (a+bx)^{17}}{17 b^{11}} + \frac{35 d^6 (bc-ad)^4 (a+bx)^{18}}{3 b^{11}} + \frac{120 d^7 (bc-ad)^3 (a+bx)^{19}}{19 b^{11}} + \\
& \frac{9 d^8 (bc-ad)^2 (a+bx)^{20}}{4 b^{11}} + \frac{10 d^9 (bc-ad) (a+bx)^{21}}{21 b^{11}} + \frac{d^{10} (a+bx)^{22}}{22 b^{11}}
\end{aligned}$$

Result (type 1, 1702 leaves):

$$\begin{aligned}
 & a^{11} c^{10} x + \frac{1}{2} a^{10} c^9 (11 b c + 10 a d) x^2 + \frac{5}{3} a^9 c^8 (11 b^2 c^2 + 22 a b c d + 9 a^2 d^2) x^3 + \\
 & \frac{5}{4} a^8 c^7 (33 b^3 c^3 + 110 a b^2 c^2 d + 99 a^2 b c d^2 + 24 a^3 d^3) x^4 + \\
 & 3 a^7 c^6 (22 b^4 c^4 + 110 a b^3 c^3 d + 165 a^2 b^2 c^2 d^2 + 88 a^3 b c d^3 + 14 a^4 d^4) x^5 + \frac{1}{2} a^6 c^5 \\
 & (154 b^5 c^5 + 1100 a b^4 c^4 d + 2475 a^2 b^3 c^3 d^2 + 2200 a^3 b^2 c^2 d^3 + 770 a^4 b c d^4 + 84 a^5 d^5) x^6 + \frac{6}{7} a^5 c^4 \\
 & (77 b^6 c^6 + 770 a b^5 c^5 d + 2475 a^2 b^4 c^4 d^2 + 3300 a^3 b^3 c^3 d^3 + 1925 a^4 b^2 c^2 d^4 + 462 a^5 b c d^5 + 35 a^6 d^6) \\
 & x^7 + \frac{15}{4} a^4 c^3 (11 b^7 c^7 + 154 a b^6 c^6 d + 693 a^2 b^5 c^5 d^2 + \\
 & 1320 a^3 b^4 c^4 d^3 + 1155 a^4 b^3 c^3 d^4 + 462 a^5 b^2 c^2 d^5 + 77 a^6 b c d^6 + 4 a^7 d^7) x^8 + \\
 & \frac{5}{3} a^3 c^2 (11 b^8 c^8 + 220 a b^7 c^7 d + 1386 a^2 b^6 c^6 d^2 + 3696 a^3 b^5 c^5 d^3 + 4620 a^4 b^4 c^4 d^4 + \\
 & 2772 a^5 b^3 c^3 d^5 + 770 a^6 b^2 c^2 d^6 + 88 a^7 b c d^7 + 3 a^8 d^8) x^9 + \\
 & \frac{1}{2} a^2 c (11 b^9 c^9 + 330 a b^8 c^8 d + 2970 a^2 b^7 c^7 d^2 + 11088 a^3 b^6 c^6 d^3 + 19404 a^4 b^5 c^5 d^4 + \\
 & 16632 a^5 b^4 c^4 d^5 + 6930 a^6 b^3 c^3 d^6 + 1320 a^7 b^2 c^2 d^7 + 99 a^8 b c d^8 + 2 a^9 d^9) x^{10} + \frac{1}{11} a \\
 & (11 b^{10} c^{10} + 550 a b^9 c^9 d + 7425 a^2 b^8 c^8 d^2 + 39600 a^3 b^7 c^7 d^3 + 97020 a^4 b^6 c^6 d^4 + 116424 a^5 b^5 c^5 d^5 + \\
 & 69300 a^6 b^4 c^4 d^6 + 19800 a^7 b^3 c^3 d^7 + 2475 a^8 b^2 c^2 d^8 + 110 a^9 b c d^9 + a^{10} d^{10}) x^{11} + \\
 & \frac{1}{12} b (b^{10} c^{10} + 110 a b^9 c^9 d + 2475 a^2 b^8 c^8 d^2 + 19800 a^3 b^7 c^7 d^3 + 69300 a^4 b^6 c^6 d^4 + 116424 a^5 b^5 c^5 d^5 + \\
 & 97020 a^6 b^4 c^4 d^6 + 39600 a^7 b^3 c^3 d^7 + 7425 a^8 b^2 c^2 d^8 + 550 a^9 b c d^9 + 11 a^{10} d^{10}) x^{12} + \\
 & \frac{5}{13} b^2 d (2 b^9 c^9 + 99 a b^8 c^8 d + 1320 a^2 b^7 c^7 d^2 + 6930 a^3 b^6 c^6 d^3 + 16632 a^4 b^5 c^5 d^4 + \\
 & 19404 a^5 b^4 c^4 d^5 + 11088 a^6 b^3 c^3 d^6 + 2970 a^7 b^2 c^2 d^7 + 330 a^8 b c d^8 + 11 a^9 d^9) x^{13} + \\
 & \frac{15}{14} b^3 d^2 (3 b^8 c^8 + 88 a b^7 c^7 d + 770 a^2 b^6 c^6 d^2 + 2772 a^3 b^5 c^5 d^3 + 4620 a^4 b^4 c^4 d^4 + \\
 & 3696 a^5 b^3 c^3 d^5 + 1386 a^6 b^2 c^2 d^6 + 220 a^7 b c d^7 + 11 a^8 d^8) x^{14} + \\
 & 2 b^4 d^3 (4 b^7 c^7 + 77 a b^6 c^6 d + 462 a^2 b^5 c^5 d^2 + 1155 a^3 b^4 c^4 d^3 + 1320 a^4 b^3 c^3 d^4 + \\
 & 693 a^5 b^2 c^2 d^5 + 154 a^6 b c d^6 + 11 a^7 d^7) x^{15} + \frac{3}{8} b^5 d^4 (35 b^6 c^6 + 462 a b^5 c^5 d + \\
 & 1925 a^2 b^4 c^4 d^2 + 3300 a^3 b^3 c^3 d^3 + 2475 a^4 b^2 c^2 d^4 + 770 a^5 b c d^5 + 77 a^6 d^6) x^{16} + \\
 & \frac{3}{17} b^6 d^5 (84 b^5 c^5 + 770 a b^4 c^4 d + 2200 a^2 b^3 c^3 d^2 + 2475 a^3 b^2 c^2 d^3 + 1100 a^4 b c d^4 + 154 a^5 d^5) x^{17} + \\
 & \frac{5}{6} b^7 d^6 (14 b^4 c^4 + 88 a b^3 c^3 d + 165 a^2 b^2 c^2 d^2 + 110 a^3 b c d^3 + 22 a^4 d^4) x^{18} + \\
 & \frac{5}{19} b^8 d^7 (24 b^3 c^3 + 99 a b^2 c^2 d + 110 a^2 b c d^2 + 33 a^3 d^3) x^{19} + \\
 & \frac{1}{4} b^9 d^8 (9 b^2 c^2 + 22 a b c d + 11 a^2 d^2) x^{20} + \\
 & \frac{1}{21} b^{10} d^9 (10 b c + 11 a d) x^{21} + \frac{1}{22} b^{11} d^{10} x^{22}
 \end{aligned}$$

### Problem 1301: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^{10} (c + d x)^{10} dx$$

Optimal (type 1, 279 leaves, 2 steps):

$$\begin{aligned} & \frac{(bc - ad)^{10} (a + bx)^{11}}{11 b^{11}} + \frac{5 d (bc - ad)^9 (a + bx)^{12}}{6 b^{11}} + \\ & \frac{45 d^2 (bc - ad)^8 (a + bx)^{13}}{13 b^{11}} + \frac{60 d^3 (bc - ad)^7 (a + bx)^{14}}{7 b^{11}} + \frac{14 d^4 (bc - ad)^6 (a + bx)^{15}}{b^{11}} + \\ & \frac{63 d^5 (bc - ad)^5 (a + bx)^{16}}{4 b^{11}} + \frac{210 d^6 (bc - ad)^4 (a + bx)^{17}}{17 b^{11}} + \frac{20 d^7 (bc - ad)^3 (a + bx)^{18}}{3 b^{11}} + \\ & \frac{45 d^8 (bc - ad)^2 (a + bx)^{19}}{19 b^{11}} + \frac{d^9 (bc - ad) (a + bx)^{20}}{2 b^{11}} + \frac{d^{10} (a + bx)^{21}}{21 b^{11}} \end{aligned}$$

Result (type 1, 1539 leaves):



$$\begin{aligned}
 & a^{10} c^{10} x + 5 a^9 c^9 (b c + a d) x^2 + \frac{5}{3} a^8 c^8 (9 b^2 c^2 + 20 a b c d + 9 a^2 d^2) x^3 + \\
 & \frac{15}{2} a^7 c^7 (4 b^3 c^3 + 15 a b^2 c^2 d + 15 a^2 b c d^2 + 4 a^3 d^3) x^4 + \\
 & 3 a^6 c^6 (14 b^4 c^4 + 80 a b^3 c^3 d + 135 a^2 b^2 c^2 d^2 + 80 a^3 b c d^3 + 14 a^4 d^4) x^5 + \\
 & 2 a^5 c^5 (21 b^5 c^5 + 175 a b^4 c^4 d + 450 a^2 b^3 c^3 d^2 + 450 a^3 b^2 c^2 d^3 + 175 a^4 b c d^4 + 21 a^5 d^5) x^6 + \\
 & \frac{30}{7} a^4 c^4 (7 b^6 c^6 + 84 a b^5 c^5 d + 315 a^2 b^4 c^4 d^2 + 480 a^3 b^3 c^3 d^3 + 315 a^4 b^2 c^2 d^4 + 84 a^5 b c d^5 + 7 a^6 d^6) \\
 & x^7 + \frac{15}{2} a^3 c^3 (2 b^7 c^7 + 35 a b^6 c^6 d + 189 a^2 b^5 c^5 d^2 + \\
 & 420 a^3 b^4 c^4 d^3 + 420 a^4 b^3 c^3 d^4 + 189 a^5 b^2 c^2 d^5 + 35 a^6 b c d^6 + 2 a^7 d^7) x^8 + \\
 & \frac{5}{3} a^2 c^2 (3 b^8 c^8 + 80 a b^7 c^7 d + 630 a^2 b^6 c^6 d^2 + 2016 a^3 b^5 c^5 d^3 + 2940 a^4 b^4 c^4 d^4 + \\
 & 2016 a^5 b^3 c^3 d^5 + 630 a^6 b^2 c^2 d^6 + 80 a^7 b c d^7 + 3 a^8 d^8) x^9 + \\
 & a c (b^9 c^9 + 45 a b^8 c^8 d + 540 a^2 b^7 c^7 d^2 + 2520 a^3 b^6 c^6 d^3 + 5292 a^4 b^5 c^5 d^4 + \\
 & 5292 a^5 b^4 c^4 d^5 + 2520 a^6 b^3 c^3 d^6 + 540 a^7 b^2 c^2 d^7 + 45 a^8 b c d^8 + a^9 d^9) x^{10} + \\
 & \frac{1}{11} (b^{10} c^{10} + 100 a b^9 c^9 d + 2025 a^2 b^8 c^8 d^2 + 14400 a^3 b^7 c^7 d^3 + 44100 a^4 b^6 c^6 d^4 + 63504 a^5 b^5 c^5 d^5 + \\
 & 44100 a^6 b^4 c^4 d^6 + 14400 a^7 b^3 c^3 d^7 + 2025 a^8 b^2 c^2 d^8 + 100 a^9 b c d^9 + a^{10} d^{10}) x^{11} + \\
 & \frac{5}{6} b d (b^9 c^9 + 45 a b^8 c^8 d + 540 a^2 b^7 c^7 d^2 + 2520 a^3 b^6 c^6 d^3 + 5292 a^4 b^5 c^5 d^4 + \\
 & 5292 a^5 b^4 c^4 d^5 + 2520 a^6 b^3 c^3 d^6 + 540 a^7 b^2 c^2 d^7 + 45 a^8 b c d^8 + a^9 d^9) x^{12} + \\
 & \frac{15}{13} b^2 d^2 (3 b^8 c^8 + 80 a b^7 c^7 d + 630 a^2 b^6 c^6 d^2 + 2016 a^3 b^5 c^5 d^3 + 2940 a^4 b^4 c^4 d^4 + \\
 & 2016 a^5 b^3 c^3 d^5 + 630 a^6 b^2 c^2 d^6 + 80 a^7 b c d^7 + 3 a^8 d^8) x^{13} + \\
 & \frac{30}{7} b^3 d^3 (2 b^7 c^7 + 35 a b^6 c^6 d + 189 a^2 b^5 c^5 d^2 + 420 a^3 b^4 c^4 d^3 + 420 a^4 b^3 c^3 d^4 + \\
 & 189 a^5 b^2 c^2 d^5 + 35 a^6 b c d^6 + 2 a^7 d^7) x^{14} + 2 b^4 d^4 \\
 & (7 b^6 c^6 + 84 a b^5 c^5 d + 315 a^2 b^4 c^4 d^2 + 480 a^3 b^3 c^3 d^3 + 315 a^4 b^2 c^2 d^4 + 84 a^5 b c d^5 + 7 a^6 d^6) x^{15} + \\
 & \frac{3}{4} b^5 d^5 (21 b^5 c^5 + 175 a b^4 c^4 d + 450 a^2 b^3 c^3 d^2 + 450 a^3 b^2 c^2 d^3 + 175 a^4 b c d^4 + 21 a^5 d^5) x^{16} + \\
 & \frac{15}{17} b^6 d^6 (14 b^4 c^4 + 80 a b^3 c^3 d + 135 a^2 b^2 c^2 d^2 + 80 a^3 b c d^3 + 14 a^4 d^4) x^{17} + \\
 & \frac{5}{3} b^7 d^7 (4 b^3 c^3 + 15 a b^2 c^2 d + 15 a^2 b c d^2 + 4 a^3 d^3) x^{18} + \\
 & \frac{5}{19} b^8 d^8 (9 b^2 c^2 + 20 a b c d + 9 a^2 d^2) x^{19} + \frac{1}{2} b^9 d^9 (b c + a d) x^{20} + \frac{1}{21} b^{10} d^{10} x^{21}
 \end{aligned}$$

**Problem 1302: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^9 (c + d x)^{10} dx$$

Optimal (type 1, 250 leaves, 2 steps):

$$\begin{aligned}
& - \frac{(bc-ad)^9 (c+dx)^{11}}{11 d^{10}} + \frac{3b (bc-ad)^8 (c+dx)^{12}}{4 d^{10}} - \frac{36b^2 (bc-ad)^7 (c+dx)^{13}}{13 d^{10}} + \\
& \frac{6b^3 (bc-ad)^6 (c+dx)^{14}}{d^{10}} - \frac{42b^4 (bc-ad)^5 (c+dx)^{15}}{5 d^{10}} + \frac{63b^5 (bc-ad)^4 (c+dx)^{16}}{8 d^{10}} - \\
& \frac{84b^6 (bc-ad)^3 (c+dx)^{17}}{17 d^{10}} + \frac{2b^7 (bc-ad)^2 (c+dx)^{18}}{d^{10}} - \frac{9b^8 (bc-ad) (c+dx)^{19}}{19 d^{10}} + \frac{b^9 (c+dx)^{20}}{20 d^{10}}
\end{aligned}$$

Result (type 1, 1397 leaves):

$$\begin{aligned}
& a^9 c^{10} x + \frac{1}{2} a^8 c^9 (9bc + 10ad) x^2 + 3a^7 c^8 (4b^2 c^2 + 10ab cd + 5a^2 d^2) x^3 + \\
& \frac{3}{4} a^6 c^7 (28b^3 c^3 + 120a b^2 c^2 d + 135a^2 b c d^2 + 40a^3 d^3) x^4 + \\
& \frac{6}{5} a^5 c^6 (21b^4 c^4 + 140a b^3 c^3 d + 270a^2 b^2 c^2 d^2 + 180a^3 b c d^3 + 35a^4 d^4) x^5 + \\
& 3a^4 c^5 (7b^5 c^5 + 70a b^4 c^4 d + 210a^2 b^3 c^3 d^2 + 240a^3 b^2 c^2 d^3 + 105a^4 b c d^4 + 14a^5 d^5) x^6 + \\
& 6a^3 c^4 (2b^6 c^6 + 30a b^5 c^5 d + 135a^2 b^4 c^4 d^2 + 240a^3 b^3 c^3 d^3 + 180a^4 b^2 c^2 d^4 + 54a^5 b c d^5 + 5a^6 d^6) x^7 + \\
& \frac{3}{4} a^2 c^3 (6b^7 c^7 + 140a b^6 c^6 d + 945a^2 b^5 c^5 d^2 + 2520a^3 b^4 c^4 d^3 + \\
& \quad 2940a^4 b^3 c^3 d^4 + 1512a^5 b^2 c^2 d^5 + 315a^6 b c d^6 + 20a^7 d^7) x^8 + \\
& a c^2 (b^8 c^8 + 40a b^7 c^7 d + 420a^2 b^6 c^6 d^2 + 1680a^3 b^5 c^5 d^3 + 2940a^4 b^4 c^4 d^4 + \\
& \quad 2352a^5 b^3 c^3 d^5 + 840a^6 b^2 c^2 d^6 + 120a^7 b c d^7 + 5a^8 d^8) x^9 + \\
& \frac{1}{10} c (b^9 c^9 + 90a b^8 c^8 d + 1620a^2 b^7 c^7 d^2 + 10080a^3 b^6 c^6 d^3 + 26460a^4 b^5 c^5 d^4 + \\
& \quad 31752a^5 b^4 c^4 d^5 + 17640a^6 b^3 c^3 d^6 + 4320a^7 b^2 c^2 d^7 + 405a^8 b c d^8 + 10a^9 d^9) x^{10} + \\
& \frac{1}{11} d (10b^9 c^9 + 405a b^8 c^8 d + 4320a^2 b^7 c^7 d^2 + 17640a^3 b^6 c^6 d^3 + 31752a^4 b^5 c^5 d^4 + \\
& \quad 26460a^5 b^4 c^4 d^5 + 10080a^6 b^3 c^3 d^6 + 1620a^7 b^2 c^2 d^7 + 90a^8 b c d^8 + a^9 d^9) x^{11} + \\
& \frac{3}{4} b d^2 (5b^8 c^8 + 120a b^7 c^7 d + 840a^2 b^6 c^6 d^2 + 2352a^3 b^5 c^5 d^3 + 2940a^4 b^4 c^4 d^4 + \\
& \quad 1680a^5 b^3 c^3 d^5 + 420a^6 b^2 c^2 d^6 + 40a^7 b c d^7 + a^8 d^8) x^{12} + \\
& \frac{6}{13} b^2 d^3 (20b^7 c^7 + 315a b^6 c^6 d + 1512a^2 b^5 c^5 d^2 + 2940a^3 b^4 c^4 d^3 + \\
& \quad 2520a^4 b^3 c^3 d^4 + 945a^5 b^2 c^2 d^5 + 140a^6 b c d^6 + 6a^7 d^7) x^{13} + 3b^3 d^4 \\
& (5b^6 c^6 + 54a b^5 c^5 d + 180a^2 b^4 c^4 d^2 + 240a^3 b^3 c^3 d^3 + 135a^4 b^2 c^2 d^4 + 30a^5 b c d^5 + 2a^6 d^6) x^{14} + \\
& \frac{6}{5} b^4 d^5 (14b^5 c^5 + 105a b^4 c^4 d + 240a^2 b^3 c^3 d^2 + 210a^3 b^2 c^2 d^3 + 70a^4 b c d^4 + 7a^5 d^5) x^{15} + \\
& \frac{3}{8} b^5 d^6 (35b^4 c^4 + 180a b^3 c^3 d + 270a^2 b^2 c^2 d^2 + 140a^3 b c d^3 + 21a^4 d^4) x^{16} + \\
& \frac{3}{17} b^6 d^7 (40b^3 c^3 + 135a b^2 c^2 d + 120a^2 b c d^2 + 28a^3 d^3) x^{17} + \\
& \frac{1}{2} b^7 d^8 (5b^2 c^2 + 10a b c d + 4a^2 d^2) x^{18} + \frac{1}{19} b^8 d^9 (10b c + 9a d) x^{19} + \frac{1}{20} b^9 d^{10} x^{20}
\end{aligned}$$

### Problem 1303: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^8 (c + d x)^{10} dx$$

Optimal (type 1, 225 leaves, 2 steps):

$$\frac{(b c - a d)^8 (c + d x)^{11}}{11 d^9} - \frac{2 b (b c - a d)^7 (c + d x)^{12}}{3 d^9} + \frac{28 b^2 (b c - a d)^6 (c + d x)^{13}}{13 d^9} - \frac{4 b^3 (b c - a d)^5 (c + d x)^{14}}{d^9} + \frac{14 b^4 (b c - a d)^4 (c + d x)^{15}}{3 d^9} - \frac{7 b^5 (b c - a d)^3 (c + d x)^{16}}{2 d^9} + \frac{28 b^6 (b c - a d)^2 (c + d x)^{17}}{17 d^9} - \frac{4 b^7 (b c - a d) (c + d x)^{18}}{9 d^9} + \frac{b^8 (c + d x)^{19}}{19 d^9}$$

Result (type 1, 1241 leaves):

$$\begin{aligned} & a^8 c^{10} x + a^7 c^9 (4 b c + 5 a d) x^2 + \frac{1}{3} a^6 c^8 (28 b^2 c^2 + 80 a b c d + 45 a^2 d^2) x^3 + \\ & 2 a^5 c^7 (7 b^3 c^3 + 35 a b^2 c^2 d + 45 a^2 b c d^2 + 15 a^3 d^3) x^4 + \\ & 2 a^4 c^6 (7 b^4 c^4 + 56 a b^3 c^3 d + 126 a^2 b^2 c^2 d^2 + 96 a^3 b c d^3 + 21 a^4 d^4) x^5 + \\ & \frac{14}{3} a^3 c^5 (2 b^5 c^5 + 25 a b^4 c^4 d + 90 a^2 b^3 c^3 d^2 + 120 a^3 b^2 c^2 d^3 + 60 a^4 b c d^4 + 9 a^5 d^5) x^6 + \\ & 2 a^2 c^4 (2 b^6 c^6 + 40 a b^5 c^5 d + 225 a^2 b^4 c^4 d^2 + 480 a^3 b^3 c^3 d^3 + 420 a^4 b^2 c^2 d^4 + 144 a^5 b c d^5 + 15 a^6 d^6) \\ & x^7 + a c^3 (b^7 c^7 + 35 a b^6 c^6 d + 315 a^2 b^5 c^5 d^2 + 1050 a^3 b^4 c^4 d^3 + \\ & 1470 a^4 b^3 c^3 d^4 + 882 a^5 b^2 c^2 d^5 + 210 a^6 b c d^6 + 15 a^7 d^7) x^8 + \\ & \frac{1}{9} c^2 (b^8 c^8 + 80 a b^7 c^7 d + 1260 a^2 b^6 c^6 d^2 + 6720 a^3 b^5 c^5 d^3 + 14700 a^4 b^4 c^4 d^4 + \\ & 14112 a^5 b^3 c^3 d^5 + 5880 a^6 b^2 c^2 d^6 + 960 a^7 b c d^7 + 45 a^8 d^8) x^9 + \\ & c d (b^8 c^8 + 36 a b^7 c^7 d + 336 a^2 b^6 c^6 d^2 + 1176 a^3 b^5 c^5 d^3 + 1764 a^4 b^4 c^4 d^4 + \\ & 1176 a^5 b^3 c^3 d^5 + 336 a^6 b^2 c^2 d^6 + 36 a^7 b c d^7 + a^8 d^8) x^{10} + \\ & \frac{1}{11} d^2 (45 b^8 c^8 + 960 a b^7 c^7 d + 5880 a^2 b^6 c^6 d^2 + 14112 a^3 b^5 c^5 d^3 + 14700 a^4 b^4 c^4 d^4 + \\ & 6720 a^5 b^3 c^3 d^5 + 1260 a^6 b^2 c^2 d^6 + 80 a^7 b c d^7 + a^8 d^8) x^{11} + \\ & \frac{2}{3} b d^3 (15 b^7 c^7 + 210 a b^6 c^6 d + 882 a^2 b^5 c^5 d^2 + 1470 a^3 b^4 c^4 d^3 + 1050 a^4 b^3 c^3 d^4 + \\ & 315 a^5 b^2 c^2 d^5 + 35 a^6 b c d^6 + a^7 d^7) x^{12} + \frac{14}{13} b^2 d^4 \\ & (15 b^6 c^6 + 144 a b^5 c^5 d + 420 a^2 b^4 c^4 d^2 + 480 a^3 b^3 c^3 d^3 + 225 a^4 b^2 c^2 d^4 + 40 a^5 b c d^5 + 2 a^6 d^6) x^{13} + \\ & 2 b^3 d^5 (9 b^5 c^5 + 60 a b^4 c^4 d + 120 a^2 b^3 c^3 d^2 + 90 a^3 b^2 c^2 d^3 + 25 a^4 b c d^4 + 2 a^5 d^5) x^{14} + \\ & \frac{2}{3} b^4 d^6 (21 b^4 c^4 + 96 a b^3 c^3 d + 126 a^2 b^2 c^2 d^2 + 56 a^3 b c d^3 + 7 a^4 d^4) x^{15} + \\ & \frac{1}{2} b^5 d^7 (15 b^3 c^3 + 45 a b^2 c^2 d + 35 a^2 b c d^2 + 7 a^3 d^3) x^{16} + \\ & \frac{1}{17} b^6 d^8 (45 b^2 c^2 + 80 a b c d + 28 a^2 d^2) x^{17} + \frac{1}{9} b^7 d^9 (5 b c + 4 a d) x^{18} + \frac{1}{19} b^8 d^{10} x^{19} \end{aligned}$$

### Problem 1304: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^7 (c + d x)^{10} dx$$

Optimal (type 1, 200 leaves, 2 steps):

$$\begin{aligned} & -\frac{(bc - ad)^7 (c + dx)^{11}}{11 d^8} + \frac{7b (bc - ad)^6 (c + dx)^{12}}{12 d^8} - \\ & \frac{21 b^2 (bc - ad)^5 (c + dx)^{13}}{13 d^8} + \frac{5 b^3 (bc - ad)^4 (c + dx)^{14}}{2 d^8} - \frac{7 b^4 (bc - ad)^3 (c + dx)^{15}}{3 d^8} + \\ & \frac{21 b^5 (bc - ad)^2 (c + dx)^{16}}{16 d^8} - \frac{7 b^6 (bc - ad) (c + dx)^{17}}{17 d^8} + \frac{b^7 (c + dx)^{18}}{18 d^8} \end{aligned}$$

Result (type 1, 1105 leaves):

$$\begin{aligned} & a^7 c^{10} x + \frac{1}{2} a^6 c^9 (7 b c + 10 a d) x^2 + \frac{1}{3} a^5 c^8 (21 b^2 c^2 + 70 a b c d + 45 a^2 d^2) x^3 + \\ & \frac{5}{4} a^4 c^7 (7 b^3 c^3 + 42 a b^2 c^2 d + 63 a^2 b c d^2 + 24 a^3 d^3) x^4 + \\ & 7 a^3 c^6 (b^4 c^4 + 10 a b^3 c^3 d + 27 a^2 b^2 c^2 d^2 + 24 a^3 b c d^3 + 6 a^4 d^4) x^5 + \\ & \frac{7}{6} a^2 c^5 (3 b^5 c^5 + 50 a b^4 c^4 d + 225 a^2 b^3 c^3 d^2 + 360 a^3 b^2 c^2 d^3 + 210 a^4 b c d^4 + 36 a^5 d^5) x^6 + \\ & a c^4 (b^6 c^6 + 30 a b^5 c^5 d + 225 a^2 b^4 c^4 d^2 + 600 a^3 b^3 c^3 d^3 + 630 a^4 b^2 c^2 d^4 + 252 a^5 b c d^5 + 30 a^6 d^6) x^7 + \\ & \frac{1}{8} c^3 (b^7 c^7 + 70 a b^6 c^6 d + 945 a^2 b^5 c^5 d^2 + 4200 a^3 b^4 c^4 d^3 + \\ & \quad 7350 a^4 b^3 c^3 d^4 + 5292 a^5 b^2 c^2 d^5 + 1470 a^6 b c d^6 + 120 a^7 d^7) x^8 + \\ & \frac{5}{9} c^2 d (2 b^7 c^7 + 63 a b^6 c^6 d + 504 a^2 b^5 c^5 d^2 + 1470 a^3 b^4 c^4 d^3 + 1764 a^4 b^3 c^3 d^4 + \\ & \quad 882 a^5 b^2 c^2 d^5 + 168 a^6 b c d^6 + 9 a^7 d^7) x^9 + \frac{1}{2} c d^2 (9 b^7 c^7 + 168 a b^6 c^6 d + 882 a^2 b^5 c^5 d^2 + \\ & \quad 1764 a^3 b^4 c^4 d^3 + 1470 a^4 b^3 c^3 d^4 + 504 a^5 b^2 c^2 d^5 + 63 a^6 b c d^6 + 2 a^7 d^7) x^{10} + \\ & \frac{1}{11} d^3 (120 b^7 c^7 + 1470 a b^6 c^6 d + 5292 a^2 b^5 c^5 d^2 + 7350 a^3 b^4 c^4 d^3 + \\ & \quad 4200 a^4 b^3 c^3 d^4 + 945 a^5 b^2 c^2 d^5 + 70 a^6 b c d^6 + a^7 d^7) x^{11} + \frac{7}{12} b d^4 \\ & (30 b^6 c^6 + 252 a b^5 c^5 d + 630 a^2 b^4 c^4 d^2 + 600 a^3 b^3 c^3 d^3 + 225 a^4 b^2 c^2 d^4 + 30 a^5 b c d^5 + a^6 d^6) x^{12} + \\ & \frac{7}{13} b^2 d^5 (36 b^5 c^5 + 210 a b^4 c^4 d + 360 a^2 b^3 c^3 d^2 + 225 a^3 b^2 c^2 d^3 + 50 a^4 b c d^4 + 3 a^5 d^5) x^{13} + \\ & \frac{5}{2} b^3 d^6 (6 b^4 c^4 + 24 a b^3 c^3 d + 27 a^2 b^2 c^2 d^2 + 10 a^3 b c d^3 + a^4 d^4) x^{14} + \\ & \frac{1}{3} b^4 d^7 (24 b^3 c^3 + 63 a b^2 c^2 d + 42 a^2 b c d^2 + 7 a^3 d^3) x^{15} + \\ & \frac{1}{16} b^5 d^8 (45 b^2 c^2 + 70 a b c d + 21 a^2 d^2) x^{16} + \frac{1}{17} b^6 d^9 (10 b c + 7 a d) x^{17} + \frac{1}{18} b^7 d^{10} x^{18} \end{aligned}$$

### Problem 1305: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^6 (c + d x)^{10} dx$$

Optimal (type 1, 170 leaves, 2 steps):

$$\frac{(b c - a d)^6 (c + d x)^{11}}{11 d^7} - \frac{b (b c - a d)^5 (c + d x)^{12}}{2 d^7} + \frac{15 b^2 (b c - a d)^4 (c + d x)^{13}}{13 d^7} - \frac{10 b^3 (b c - a d)^3 (c + d x)^{14}}{7 d^7} + \frac{b^4 (b c - a d)^2 (c + d x)^{15}}{d^7} - \frac{3 b^5 (b c - a d) (c + d x)^{16}}{8 d^7} + \frac{b^6 (c + d x)^{17}}{17 d^7}$$

Result (type 1, 939 leaves):

$$\begin{aligned} & a^6 c^{10} x + a^5 c^9 (3 b c + 5 a d) x^2 + 5 a^4 c^8 (b^2 c^2 + 4 a b c d + 3 a^2 d^2) x^3 + \\ & \frac{5}{2} a^3 c^7 (2 b^3 c^3 + 15 a b^2 c^2 d + 27 a^2 b c d^2 + 12 a^3 d^3) x^4 + \\ & a^2 c^6 (3 b^4 c^4 + 40 a b^3 c^3 d + 135 a^2 b^2 c^2 d^2 + 144 a^3 b c d^3 + 42 a^4 d^4) x^5 + \\ & a c^5 (b^5 c^5 + 25 a b^4 c^4 d + 150 a^2 b^3 c^3 d^2 + 300 a^3 b^2 c^2 d^3 + 210 a^4 b c d^4 + 42 a^5 d^5) x^6 + \frac{1}{7} c^4 \\ & (b^6 c^6 + 60 a b^5 c^5 d + 675 a^2 b^4 c^4 d^2 + 2400 a^3 b^3 c^3 d^3 + 3150 a^4 b^2 c^2 d^4 + 1512 a^5 b c d^5 + 210 a^6 d^6) x^7 + \\ & \frac{5}{4} c^3 d (b^6 c^6 + 27 a b^5 c^5 d + 180 a^2 b^4 c^4 d^2 + 420 a^3 b^3 c^3 d^3 + 378 a^4 b^2 c^2 d^4 + 126 a^5 b c d^5 + 12 a^6 d^6) x^8 + \\ & 5 c^2 d^2 (b^6 c^6 + 16 a b^5 c^5 d + 70 a^2 b^4 c^4 d^2 + 112 a^3 b^3 c^3 d^3 + 70 a^4 b^2 c^2 d^4 + 16 a^5 b c d^5 + a^6 d^6) x^9 + \\ & c d^3 (12 b^6 c^6 + 126 a b^5 c^5 d + 378 a^2 b^4 c^4 d^2 + 420 a^3 b^3 c^3 d^3 + 180 a^4 b^2 c^2 d^4 + 27 a^5 b c d^5 + a^6 d^6) x^{10} + \\ & \frac{1}{11} d^4 (210 b^6 c^6 + 1512 a b^5 c^5 d + 3150 a^2 b^4 c^4 d^2 + 2400 a^3 b^3 c^3 d^3 + 675 a^4 b^2 c^2 d^4 + 60 a^5 b c d^5 + a^6 d^6) \\ & x^{11} + \frac{1}{2} b d^5 (42 b^5 c^5 + 210 a b^4 c^4 d + 300 a^2 b^3 c^3 d^2 + 150 a^3 b^2 c^2 d^3 + 25 a^4 b c d^4 + a^5 d^5) x^{12} + \\ & \frac{5}{13} b^2 d^6 (42 b^4 c^4 + 144 a b^3 c^3 d + 135 a^2 b^2 c^2 d^2 + 40 a^3 b c d^3 + 3 a^4 d^4) x^{13} + \\ & \frac{5}{7} b^3 d^7 (12 b^3 c^3 + 27 a b^2 c^2 d + 15 a^2 b c d^2 + 2 a^3 d^3) x^{14} + \\ & b^4 d^8 (3 b^2 c^2 + 4 a b c d + a^2 d^2) x^{15} + \frac{1}{8} b^5 d^9 (5 b c + 3 a d) x^{16} + \frac{1}{17} b^6 d^{10} x^{17} \end{aligned}$$

### Problem 1306: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^5 (c + d x)^{10} dx$$

Optimal (type 1, 146 leaves, 2 steps):

$$-\frac{(b c - a d)^5 (c + d x)^{11}}{11 d^6} + \frac{5 b (b c - a d)^4 (c + d x)^{12}}{12 d^6} - \frac{10 b^2 (b c - a d)^3 (c + d x)^{13}}{13 d^6} + \frac{5 b^3 (b c - a d)^2 (c + d x)^{14}}{7 d^6} - \frac{b^4 (b c - a d) (c + d x)^{15}}{3 d^6} + \frac{b^5 (c + d x)^{16}}{16 d^6}$$

Result (type 1, 811 leaves):

$$\begin{aligned}
& a^5 c^{10} x + \frac{5}{2} a^4 c^9 (b c + 2 a d) x^2 + \frac{5}{3} a^3 c^8 (2 b^2 c^2 + 10 a b c d + 9 a^2 d^2) x^3 + \\
& \frac{5}{4} a^2 c^7 (2 b^3 c^3 + 20 a b^2 c^2 d + 45 a^2 b c d^2 + 24 a^3 d^3) x^4 + \\
& a c^6 (b^4 c^4 + 20 a b^3 c^3 d + 90 a^2 b^2 c^2 d^2 + 120 a^3 b c d^3 + 42 a^4 d^4) x^5 + \\
& \frac{1}{6} c^5 (b^5 c^5 + 50 a b^4 c^4 d + 450 a^2 b^3 c^3 d^2 + 1200 a^3 b^2 c^2 d^3 + 1050 a^4 b c d^4 + 252 a^5 d^5) x^6 + \\
& \frac{5}{7} c^4 d (2 b^5 c^5 + 45 a b^4 c^4 d + 240 a^2 b^3 c^3 d^2 + 420 a^3 b^2 c^2 d^3 + 252 a^4 b c d^4 + 42 a^5 d^5) x^7 + \\
& \frac{15}{8} c^3 d^2 (3 b^5 c^5 + 40 a b^4 c^4 d + 140 a^2 b^3 c^3 d^2 + 168 a^3 b^2 c^2 d^3 + 70 a^4 b c d^4 + 8 a^5 d^5) x^8 + \\
& \frac{5}{3} c^2 d^3 (8 b^5 c^5 + 70 a b^4 c^4 d + 168 a^2 b^3 c^3 d^2 + 140 a^3 b^2 c^2 d^3 + 40 a^4 b c d^4 + 3 a^5 d^5) x^9 + \\
& \frac{1}{2} c d^4 (42 b^5 c^5 + 252 a b^4 c^4 d + 420 a^2 b^3 c^3 d^2 + 240 a^3 b^2 c^2 d^3 + 45 a^4 b c d^4 + 2 a^5 d^5) x^{10} + \\
& \frac{1}{11} d^5 (252 b^5 c^5 + 1050 a b^4 c^4 d + 1200 a^2 b^3 c^3 d^2 + 450 a^3 b^2 c^2 d^3 + 50 a^4 b c d^4 + a^5 d^5) x^{11} + \\
& \frac{5}{12} b d^6 (42 b^4 c^4 + 120 a b^3 c^3 d + 90 a^2 b^2 c^2 d^2 + 20 a^3 b c d^3 + a^4 d^4) x^{12} + \\
& \frac{5}{13} b^2 d^7 (24 b^3 c^3 + 45 a b^2 c^2 d + 20 a^2 b c d^2 + 2 a^3 d^3) x^{13} + \\
& \frac{5}{14} b^3 d^8 (9 b^2 c^2 + 10 a b c d + 2 a^2 d^2) x^{14} + \frac{1}{3} b^4 d^9 (2 b c + a d) x^{15} + \frac{1}{16} b^5 d^{10} x^{16}
\end{aligned}$$

### Problem 1307: Result more than twice size of optimal antiderivative.

$$\int (a + b x)^4 (c + d x)^{10} dx$$

Optimal (type 1, 119 leaves, 2 steps):

$$\begin{aligned}
& \frac{(b c - a d)^4 (c + d x)^{11}}{11 d^5} - \frac{b (b c - a d)^3 (c + d x)^{12}}{3 d^5} + \\
& \frac{6 b^2 (b c - a d)^2 (c + d x)^{13}}{13 d^5} - \frac{2 b^3 (b c - a d) (c + d x)^{14}}{7 d^5} + \frac{b^4 (c + d x)^{15}}{15 d^5}
\end{aligned}$$

Result (type 1, 660 leaves):

$$\begin{aligned}
 & a^4 c^{10} x + a^3 c^9 (2 b c + 5 a d) x^2 + \frac{1}{3} a^2 c^8 (6 b^2 c^2 + 40 a b c d + 45 a^2 d^2) x^3 + \\
 & a c^7 (b^3 c^3 + 15 a b^2 c^2 d + 45 a^2 b c d^2 + 30 a^3 d^3) x^4 + \\
 & \frac{1}{5} c^6 (b^4 c^4 + 40 a b^3 c^3 d + 270 a^2 b^2 c^2 d^2 + 480 a^3 b c d^3 + 210 a^4 d^4) x^5 + \\
 & \frac{1}{3} c^5 d (5 b^4 c^4 + 90 a b^3 c^3 d + 360 a^2 b^2 c^2 d^2 + 420 a^3 b c d^3 + 126 a^4 d^4) x^6 + \\
 & \frac{3}{7} c^4 d^2 (15 b^4 c^4 + 160 a b^3 c^3 d + 420 a^2 b^2 c^2 d^2 + 336 a^3 b c d^3 + 70 a^4 d^4) x^7 + \\
 & 3 c^3 d^3 (5 b^4 c^4 + 35 a b^3 c^3 d + 63 a^2 b^2 c^2 d^2 + 35 a^3 b c d^3 + 5 a^4 d^4) x^8 + \\
 & \frac{1}{3} c^2 d^4 (70 b^4 c^4 + 336 a b^3 c^3 d + 420 a^2 b^2 c^2 d^2 + 160 a^3 b c d^3 + 15 a^4 d^4) x^9 + \\
 & \frac{1}{5} c d^5 (126 b^4 c^4 + 420 a b^3 c^3 d + 360 a^2 b^2 c^2 d^2 + 90 a^3 b c d^3 + 5 a^4 d^4) x^{10} + \\
 & \frac{1}{11} d^6 (210 b^4 c^4 + 480 a b^3 c^3 d + 270 a^2 b^2 c^2 d^2 + 40 a^3 b c d^3 + a^4 d^4) x^{11} + \\
 & \frac{1}{3} b d^7 (30 b^3 c^3 + 45 a b^2 c^2 d + 15 a^2 b c d^2 + a^3 d^3) x^{12} + \\
 & \frac{1}{13} b^2 d^8 (45 b^2 c^2 + 40 a b c d + 6 a^2 d^2) x^{13} + \frac{1}{7} b^3 d^9 (5 b c + 2 a d) x^{14} + \frac{1}{15} b^4 d^{10} x^{15}
 \end{aligned}$$

**Problem 1308: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^3 (c + d x)^{10} dx$$

Optimal (type 1, 92 leaves, 2 steps):

$$-\frac{(b c - a d)^3 (c + d x)^{11}}{11 d^4} + \frac{b (b c - a d)^2 (c + d x)^{12}}{4 d^4} - \frac{3 b^2 (b c - a d) (c + d x)^{13}}{13 d^4} + \frac{b^3 (c + d x)^{14}}{14 d^4}$$

Result (type 1, 511 leaves):

$$\begin{aligned}
& a^3 c^{10} x + \frac{1}{2} a^2 c^9 (3 b c + 10 a d) x^2 + a c^8 (b^2 c^2 + 10 a b c d + 15 a^2 d^2) x^3 + \\
& \frac{1}{4} c^7 (b^3 c^3 + 30 a b^2 c^2 d + 135 a^2 b c d^2 + 120 a^3 d^3) x^4 + \\
& c^6 d (2 b^3 c^3 + 27 a b^2 c^2 d + 72 a^2 b c d^2 + 42 a^3 d^3) x^5 + \\
& \frac{3}{2} c^5 d^2 (5 b^3 c^3 + 40 a b^2 c^2 d + 70 a^2 b c d^2 + 28 a^3 d^3) x^6 + \\
& \frac{6}{7} c^4 d^3 (20 b^3 c^3 + 105 a b^2 c^2 d + 126 a^2 b c d^2 + 35 a^3 d^3) x^7 + \\
& \frac{3}{4} c^3 d^4 (35 b^3 c^3 + 126 a b^2 c^2 d + 105 a^2 b c d^2 + 20 a^3 d^3) x^8 + \\
& c^2 d^5 (28 b^3 c^3 + 70 a b^2 c^2 d + 40 a^2 b c d^2 + 5 a^3 d^3) x^9 + \\
& \frac{1}{2} c d^6 (42 b^3 c^3 + 72 a b^2 c^2 d + 27 a^2 b c d^2 + 2 a^3 d^3) x^{10} + \\
& \frac{1}{11} d^7 (120 b^3 c^3 + 135 a b^2 c^2 d + 30 a^2 b c d^2 + a^3 d^3) x^{11} + \\
& \frac{1}{4} b d^8 (15 b^2 c^2 + 10 a b c d + a^2 d^2) x^{12} + \frac{1}{13} b^2 d^9 (10 b c + 3 a d) x^{13} + \frac{1}{14} b^3 d^{10} x^{14}
\end{aligned}$$

**Problem 1309: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^2 (c + d x)^{10} dx$$

Optimal (type 1, 65 leaves, 2 steps):

$$\frac{(b c - a d)^2 (c + d x)^{11}}{11 d^3} - \frac{b (b c - a d) (c + d x)^{12}}{6 d^3} + \frac{b^2 (c + d x)^{13}}{13 d^3}$$

Result (type 1, 358 leaves):

$$\begin{aligned}
& a^2 c^{10} x + a c^9 (b c + 5 a d) x^2 + \\
& \frac{1}{3} c^8 (b^2 c^2 + 20 a b c d + 45 a^2 d^2) x^3 + \frac{5}{2} c^7 d (b^2 c^2 + 9 a b c d + 12 a^2 d^2) x^4 + \\
& 3 c^6 d^2 (3 b^2 c^2 + 16 a b c d + 14 a^2 d^2) x^5 + 2 c^5 d^3 (10 b^2 c^2 + 35 a b c d + 21 a^2 d^2) x^6 + \\
& 6 c^4 d^4 (5 b^2 c^2 + 12 a b c d + 5 a^2 d^2) x^7 + \frac{3}{2} c^3 d^5 (21 b^2 c^2 + 35 a b c d + 10 a^2 d^2) x^8 + \\
& \frac{5}{3} c^2 d^6 (14 b^2 c^2 + 16 a b c d + 3 a^2 d^2) x^9 + c d^7 (12 b^2 c^2 + 9 a b c d + a^2 d^2) x^{10} + \\
& \frac{1}{11} d^8 (45 b^2 c^2 + 20 a b c d + a^2 d^2) x^{11} + \frac{1}{6} b d^9 (5 b c + a d) x^{12} + \frac{1}{13} b^2 d^{10} x^{13}
\end{aligned}$$

**Problem 1310: Result more than twice size of optimal antiderivative.**

$$\int (a + b x) (c + d x)^{10} dx$$

Optimal (type 1, 38 leaves, 2 steps):



$$-\frac{(bc-ad)(c+dx)^{11}}{11d^2} + \frac{b(c+dx)^{12}}{12d^2}$$

Result (type 1, 220 leaves):

$$\begin{aligned} & a c^{10} x + \frac{1}{2} c^9 (bc + 10ad) x^2 + \frac{5}{3} c^8 d (2bc + 9ad) x^3 + \frac{15}{4} c^7 d^2 (3bc + 8ad) x^4 + \\ & 6 c^6 d^3 (4bc + 7ad) x^5 + 7 c^5 d^4 (5bc + 6ad) x^6 + 6 c^4 d^5 (6bc + 5ad) x^7 + \frac{15}{4} c^3 d^6 (7bc + 4ad) x^8 + \\ & \frac{5}{3} c^2 d^7 (8bc + 3ad) x^9 + \frac{1}{2} c d^8 (9bc + 2ad) x^{10} + \frac{1}{11} d^9 (10bc + ad) x^{11} + \frac{1}{12} b d^{10} x^{12} \end{aligned}$$

**Problem 1312: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+dx)^{10}}{a+bx} dx$$

Optimal (type 3, 241 leaves, 2 steps):

$$\begin{aligned} & \frac{d(bc-ad)^9 x}{b^{10}} + \frac{(bc-ad)^8 (c+dx)^2}{2b^9} + \frac{(bc-ad)^7 (c+dx)^3}{3b^8} + \frac{(bc-ad)^6 (c+dx)^4}{4b^7} + \\ & \frac{(bc-ad)^5 (c+dx)^5}{5b^6} + \frac{(bc-ad)^4 (c+dx)^6}{6b^5} + \frac{(bc-ad)^3 (c+dx)^7}{7b^4} + \\ & \frac{(bc-ad)^2 (c+dx)^8}{8b^3} + \frac{(bc-ad)(c+dx)^9}{9b^2} + \frac{(c+dx)^{10}}{10b} + \frac{(bc-ad)^{10} \text{Log}[a+bx]}{b^{11}} \end{aligned}$$

Result (type 3, 591 leaves):

$$\begin{aligned} & \frac{1}{2520 b^{10}} dx \left( -2520 a^9 d^9 + 1260 a^8 b d^8 (20c + dx) - \right. \\ & 840 a^7 b^2 d^7 (135 c^2 + 15 c dx + d^2 x^2) + 210 a^6 b^3 d^6 (1440 c^3 + 270 c^2 dx + 40 c d^2 x^2 + 3 d^3 x^3) - \\ & 252 a^5 b^4 d^5 (2100 c^4 + 600 c^3 dx + 150 c^2 d^2 x^2 + 25 c d^3 x^3 + 2 d^4 x^4) + \\ & 210 a^4 b^5 d^4 (3024 c^5 + 1260 c^4 dx + 480 c^3 d^2 x^2 + 135 c^2 d^3 x^3 + 24 c d^4 x^4 + 2 d^5 x^5) - \\ & 120 a^3 b^6 d^3 (4410 c^6 + 2646 c^5 dx + 1470 c^4 d^2 x^2 + 630 c^3 d^3 x^3 + 189 c^2 d^4 x^4 + 35 c d^5 x^5 + 3 d^6 x^6) + \\ & 45 a^2 b^7 d^2 (6720 c^7 + 5880 c^6 dx + 4704 c^5 d^2 x^2 + 2940 c^4 d^3 x^3 + 1344 c^3 d^4 x^4 + \\ & 420 c^2 d^5 x^5 + 80 c d^6 x^6 + 7 d^7 x^7) - 10 a b^8 d (11340 c^8 + 15120 c^7 dx + 17640 c^6 d^2 x^2 + \\ & 15876 c^5 d^3 x^3 + 10584 c^4 d^4 x^4 + 5040 c^3 d^5 x^5 + 1620 c^2 d^6 x^6 + 315 c d^7 x^7 + 28 d^8 x^8) + \\ & \left. b^9 (25200 c^9 + 56700 c^8 dx + 100800 c^7 d^2 x^2 + 132300 c^6 d^3 x^3 + 127008 c^5 d^4 x^4 + 88200 c^4 d^5 x^5 + \right. \\ & \left. 43200 c^3 d^6 x^6 + 14175 c^2 d^7 x^7 + 2800 c d^8 x^8 + 252 d^9 x^9) \right) + \frac{(bc-ad)^{10} \text{Log}[a+bx]}{b^{11}} \end{aligned}$$

**Problem 1313: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c+dx)^{10}}{(a+bx)^2} dx$$

Optimal (type 3, 258 leaves, 2 steps):

$$\frac{45 d^2 (b c - a d)^8 x}{b^{10}} - \frac{(b c - a d)^{10}}{b^{11} (a + b x)} + \frac{60 d^3 (b c - a d)^7 (a + b x)^2}{b^{11}} + \frac{70 d^4 (b c - a d)^6 (a + b x)^3}{b^{11}} +$$

$$\frac{63 d^5 (b c - a d)^5 (a + b x)^4}{b^{11}} + \frac{42 d^6 (b c - a d)^4 (a + b x)^5}{b^{11}} + \frac{20 d^7 (b c - a d)^3 (a + b x)^6}{b^{11}} +$$

$$\frac{45 d^8 (b c - a d)^2 (a + b x)^7}{7 b^{11}} + \frac{5 d^9 (b c - a d) (a + b x)^8}{4 b^{11}} + \frac{d^{10} (a + b x)^9}{9 b^{11}} + \frac{10 d (b c - a d)^9 \text{Log}[a + b x]}{b^{11}}$$

Result (type 3, 708 leaves):

$$\frac{1}{252 b^{11} (a + b x)} \left( -252 a^{10} d^{10} + 252 a^9 b d^9 (10 c + 9 d x) + \right.$$

$$1260 a^8 b^2 d^8 (-9 c^2 - 16 c d x + d^2 x^2) - 420 a^7 b^3 d^7 (-72 c^3 - 189 c^2 d x + 27 c d^2 x^2 + d^3 x^3) +$$

$$210 a^6 b^4 d^6 (-252 c^4 - 864 c^3 d x + 216 c^2 d^2 x^2 + 18 c d^3 x^3 + d^4 x^4) -$$

$$126 a^5 b^5 d^5 (-504 c^5 - 2100 c^4 d x + 840 c^3 d^2 x^2 + 120 c^2 d^3 x^3 + 15 c d^4 x^4 + d^5 x^5) +$$

$$42 a^4 b^6 d^4 (-1260 c^6 - 6048 c^5 d x + 3780 c^4 d^2 x^2 + 840 c^3 d^3 x^3 + 180 c^2 d^4 x^4 + 27 c d^5 x^5 + 2 d^6 x^6) -$$

$$12 a^3 b^7 d^3 (-2520 c^7 - 13230 c^6 d x + 13230 c^5 d^2 x^2 + 4410 c^4 d^3 x^3 + 1470 c^3 d^4 x^4 +$$

$$378 c^2 d^5 x^5 + 63 c d^6 x^6 + 5 d^7 x^7) + 9 a^2 b^8 d^2 (-1260 c^8 - 6720 c^7 d x + 11760 c^6 d^2 x^2 +$$

$$5880 c^5 d^3 x^3 + 2940 c^4 d^4 x^4 + 1176 c^3 d^5 x^5 + 336 c^2 d^6 x^6 + 60 c d^7 x^7 + 5 d^8 x^8) -$$

$$a b^9 d (-2520 c^9 - 11340 c^8 d x + 45360 c^7 d^2 x^2 + 35280 c^6 d^3 x^3 + 26460 c^5 d^4 x^4 +$$

$$15876 c^4 d^5 x^5 + 7056 c^3 d^6 x^6 + 2160 c^2 d^7 x^7 + 405 c d^8 x^8 + 35 d^9 x^9) +$$

$$b^{10} (-252 c^{10} + 11340 c^8 d^2 x^2 + 15120 c^7 d^3 x^3 + 17640 c^6 d^4 x^4 + 15876 c^5 d^5 x^5 +$$

$$10584 c^4 d^6 x^6 + 5040 c^3 d^7 x^7 + 1620 c^2 d^8 x^8 + 315 c d^9 x^9 + 28 d^{10} x^{10}) -$$

$$2520 d (-b c + a d)^9 (a + b x) \text{Log}[a + b x] \Big)$$

**Problem 1314: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^{10}}{(a + b x)^3} dx$$

Optimal (type 3, 262 leaves, 2 steps):

$$\frac{120 d^3 (b c - a d)^7 x}{b^{10}} - \frac{(b c - a d)^{10}}{2 b^{11} (a + b x)^2} - \frac{10 d (b c - a d)^9}{b^{11} (a + b x)} +$$

$$\frac{105 d^4 (b c - a d)^6 (a + b x)^2}{b^{11}} + \frac{84 d^5 (b c - a d)^5 (a + b x)^3}{b^{11}} +$$

$$\frac{105 d^6 (b c - a d)^4 (a + b x)^4}{2 b^{11}} + \frac{24 d^7 (b c - a d)^3 (a + b x)^5}{b^{11}} + \frac{15 d^8 (b c - a d)^2 (a + b x)^6}{2 b^{11}} +$$

$$\frac{10 d^9 (b c - a d) (a + b x)^7}{7 b^{11}} + \frac{d^{10} (a + b x)^8}{8 b^{11}} + \frac{45 d^2 (b c - a d)^8 \text{Log}[a + b x]}{b^{11}}$$

Result (type 3, 708 leaves):

$$\frac{1}{56 b^{11} (a+b x)^2} \left( 532 a^{10} d^{10} - 56 a^9 b d^9 (85 c + 26 d x) + \right. \\
 28 a^8 b^2 d^8 (675 c^2 + 380 c d x - 116 d^2 x^2) - 280 a^7 b^3 d^7 (156 c^3 + 117 c^2 d x - 91 c d^2 x^2 + 3 d^3 x^3) + \\
 210 a^6 b^4 d^6 (308 c^4 + 256 c^3 d x - 414 c^2 d^2 x^2 + 32 c d^3 x^3 + d^4 x^4) - \\
 84 a^5 b^5 d^5 (756 c^5 + 560 c^4 d x - 2000 c^3 d^2 x^2 + 280 c^2 d^3 x^3 + 20 c d^4 x^4 + d^5 x^5) + 42 a^4 b^6 d^4 \\
 (980 c^6 + 336 c^5 d x - 4760 c^4 d^2 x^2 + 1120 c^3 d^3 x^3 + 140 c^2 d^4 x^4 + 16 c d^5 x^5 + d^6 x^6) - 24 a^3 b^7 d^3 \\
 (700 c^7 - 490 c^6 d x - 6174 c^5 d^2 x^2 + 2450 c^4 d^3 x^3 + 490 c^3 d^4 x^4 + 98 c^2 d^5 x^5 + 14 c d^6 x^6 + d^7 x^7) + \\
 3 a^2 b^8 d^2 (1260 c^8 - 4480 c^7 d x - 21560 c^6 d^2 x^2 + 15680 c^5 d^3 x^3 + \\
 4900 c^4 d^4 x^4 + 1568 c^3 d^5 x^5 + 392 c^2 d^6 x^6 + 64 c d^7 x^7 + 5 d^8 x^8) - \\
 2 a b^9 d (140 c^9 - 2520 c^8 d x - 6720 c^7 d^2 x^2 + 11760 c^6 d^3 x^3 + 5880 c^5 d^4 x^4 + \\
 2940 c^4 d^5 x^5 + 1176 c^3 d^6 x^6 + 336 c^2 d^7 x^7 + 60 c d^8 x^8 + 5 d^9 x^9) + \\
 b^{10} (-28 c^{10} - 560 c^9 d x + 6720 c^7 d^3 x^3 + 5880 c^6 d^4 x^4 + 4704 c^5 d^5 x^5 + 2940 c^4 d^6 x^6 + \\
 1344 c^3 d^7 x^7 + 420 c^2 d^8 x^8 + 80 c d^9 x^9 + 7 d^{10} x^{10}) + 2520 d^2 (b c - a d)^8 (a+b x)^2 \text{Log}[a+b x] \left. \right)$$

Problem 1320: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+d x)^{10}}{(a+b x)^9} dx$$

Optimal (type 3, 258 leaves, 2 steps):

$$\frac{d^9 (10 b c - 9 a d) x}{b^{10}} + \frac{d^{10} x^2}{2 b^9} - \frac{(b c - a d)^{10}}{8 b^{11} (a+b x)^8} - \frac{10 d (b c - a d)^9}{7 b^{11} (a+b x)^7} - \\
 \frac{15 d^2 (b c - a d)^8}{2 b^{11} (a+b x)^6} - \frac{24 d^3 (b c - a d)^7}{b^{11} (a+b x)^5} - \frac{105 d^4 (b c - a d)^6}{2 b^{11} (a+b x)^4} - \frac{84 d^5 (b c - a d)^5}{b^{11} (a+b x)^3} - \\
 \frac{105 d^6 (b c - a d)^4}{b^{11} (a+b x)^2} - \frac{120 d^7 (b c - a d)^3}{b^{11} (a+b x)} + \frac{45 d^8 (b c - a d)^2 \text{Log}[a+b x]}{b^{11}}$$

Result (type 3, 712 leaves):

$$\frac{1}{56 b^{11} (a+b x)^8} \left( 3601 a^{10} d^{10} + 2 a^9 b d^9 (-4609 c + 13144 d x) + a^8 b^2 d^8 (6849 c^2 - 68704 c d x + 81928 d^2 x^2) + \right. \\
 8 a^7 b^3 d^7 (-105 c^3 + 6534 c^2 d x - 27538 c d^2 x^2 + 17542 d^3 x^3) + \\
 14 a^6 b^4 d^6 (-15 c^4 - 480 c^3 d x + 12348 c^2 d^2 x^2 - 28112 c d^3 x^3 + 10010 d^4 x^4) - \\
 28 a^5 b^5 d^5 (3 c^5 + 60 c^4 d x + 840 c^3 d^2 x^2 - 11508 c^2 d^3 x^3 + 15050 c d^4 x^4 - 2744 d^5 x^5) - \\
 14 a^4 b^6 d^4 (3 c^6 + 48 c^5 d x + 420 c^4 d^2 x^2 + 3360 c^3 d^3 x^3 - 26250 c^2 d^4 x^4 + 19040 c d^5 x^5 - 1064 d^6 x^6) - \\
 8 a^3 b^7 d^3 (3 c^7 + 42 c^6 d x + 294 c^5 d^2 x^2 + 1470 c^4 d^3 x^3 + 7350 c^3 d^4 x^4 - 32340 c^2 d^5 x^5 + \\
 10780 c d^6 x^6 + 728 d^7 x^7) - a^2 b^8 d^2 (15 c^8 + 192 c^7 d x + 1176 c^6 d^2 x^2 + 4704 c^5 d^3 x^3 + \\
 14700 c^4 d^4 x^4 + 47040 c^3 d^5 x^5 - 105840 c^2 d^6 x^6 + 4480 c d^7 x^7 + 3248 d^8 x^8) - \\
 2 a b^9 d (5 c^9 + 60 c^8 d x + 336 c^7 d^2 x^2 + 1176 c^6 d^3 x^3 + 2940 c^5 d^4 x^4 + 5880 c^4 d^5 x^5 + \\
 11760 c^3 d^6 x^6 - 10080 c^2 d^7 x^7 - 2240 c d^8 x^8 + 140 d^9 x^9) - \\
 b^{10} (7 c^{10} + 80 c^9 d x + 420 c^8 d^2 x^2 + 1344 c^7 d^3 x^3 + 2940 c^6 d^4 x^4 + 4704 c^5 d^5 x^5 + 5880 c^4 d^6 x^6 + \\
 6720 c^3 d^7 x^7 - 560 c d^9 x^9 - 28 d^{10} x^{10}) + 2520 d^8 (b c - a d)^2 (a+b x)^8 \text{Log}[a+b x] \left. \right)$$

### Problem 1321: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+dx)^{10}}{(a+bx)^{10}} dx$$

Optimal (type 3, 257 leaves, 2 steps):

$$\begin{aligned} & \frac{d^{10} x}{b^{10}} - \frac{(bc-ad)^{10}}{9 b^{11} (a+bx)^9} - \frac{5 d (bc-ad)^9}{4 b^{11} (a+bx)^8} - \frac{45 d^2 (bc-ad)^8}{7 b^{11} (a+bx)^7} - \\ & \frac{20 d^3 (bc-ad)^7}{b^{11} (a+bx)^6} - \frac{42 d^4 (bc-ad)^6}{b^{11} (a+bx)^5} - \frac{63 d^5 (bc-ad)^5}{b^{11} (a+bx)^4} - \frac{70 d^6 (bc-ad)^4}{b^{11} (a+bx)^3} - \\ & \frac{60 d^7 (bc-ad)^3}{b^{11} (a+bx)^2} - \frac{45 d^8 (bc-ad)^2}{b^{11} (a+bx)} + \frac{10 d^9 (bc-ad) \operatorname{Log}[a+bx]}{b^{11}} \end{aligned}$$

Result (type 3, 708 leaves):

$$\begin{aligned} & - \frac{1}{252 b^{11} (a+bx)^9} \\ & \left( 4861 a^{10} d^{10} + a^9 b d^9 (-7129 c + 41229 dx) + 9 a^8 b^2 d^8 (140 c^2 - 6849 c dx + 17064 d^2 x^2) + \right. \\ & \quad 12 a^7 b^3 d^7 (35 c^3 + 945 c^2 dx - 19602 c d^2 x^2 + 27342 d^3 x^3) + \\ & \quad 42 a^6 b^4 d^6 (5 c^4 + 90 c^3 dx + 1080 c^2 d^2 x^2 - 12348 c d^3 x^3 + 10458 d^4 x^4) + \\ & \quad 126 a^5 b^5 d^5 (c^5 + 15 c^4 dx + 120 c^3 d^2 x^2 + 840 c^2 d^3 x^3 - 5754 c d^4 x^4 + 2982 d^5 x^5) + \\ & \quad 42 a^4 b^6 d^4 (2 c^6 + 27 c^5 dx + 180 c^4 d^2 x^2 + 840 c^3 d^3 x^3 + 3780 c^2 d^4 x^4 - 15750 c d^5 x^5 + 4704 d^6 x^6) + \\ & \quad 12 a^3 b^7 d^3 (5 c^7 + 63 c^6 dx + 378 c^5 d^2 x^2 + 1470 c^4 d^3 x^3 + 4410 c^3 d^4 x^4 + 13230 c^2 d^5 x^5 - \\ & \quad \quad 32340 c d^6 x^6 + 4536 d^7 x^7) + 9 a^2 b^8 d^2 (5 c^8 + 60 c^7 dx + 336 c^6 d^2 x^2 + 1176 c^5 d^3 x^3 + \\ & \quad \quad 2940 c^4 d^4 x^4 + 5880 c^3 d^5 x^5 + 11760 c^2 d^6 x^6 - 15120 c d^7 x^7 + 252 d^8 x^8) + \\ & \quad a b^9 d (35 c^9 + 405 c^8 dx + 2160 c^7 d^2 x^2 + 7056 c^6 d^3 x^3 + 15876 c^5 d^4 x^4 + 26460 c^4 d^5 x^5 + \\ & \quad \quad 35280 c^3 d^6 x^6 + 45360 c^2 d^7 x^7 - 22680 c d^8 x^8 - 2268 d^9 x^9) + b^{10} \\ & \quad \left. (28 c^{10} + 315 c^9 dx + 1620 c^8 d^2 x^2 + 5040 c^7 d^3 x^3 + 10584 c^6 d^4 x^4 + 15876 c^5 d^5 x^5 + 17640 c^4 d^6 x^6 + \right. \\ & \quad \quad \left. 15120 c^3 d^7 x^7 + 11340 c^2 d^8 x^8 - 252 d^{10} x^{10}) + 2520 d^9 (-bc+ad) (a+bx)^9 \operatorname{Log}[a+bx] \right) \end{aligned}$$

### Problem 1322: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+dx)^{10}}{(a+bx)^{11}} dx$$

Optimal (type 3, 271 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{(bc - ad)^{10}}{10 b^{11} (a + bx)^{10}} - \frac{10 d (bc - ad)^9}{9 b^{11} (a + bx)^9} - \frac{45 d^2 (bc - ad)^8}{8 b^{11} (a + bx)^8} - \\
 & \frac{120 d^3 (bc - ad)^7}{7 b^{11} (a + bx)^7} - \frac{35 d^4 (bc - ad)^6}{b^{11} (a + bx)^6} - \frac{252 d^5 (bc - ad)^5}{5 b^{11} (a + bx)^5} - \frac{105 d^6 (bc - ad)^4}{2 b^{11} (a + bx)^4} - \\
 & \frac{40 d^7 (bc - ad)^3}{b^{11} (a + bx)^3} - \frac{45 d^8 (bc - ad)^2}{2 b^{11} (a + bx)^2} - \frac{10 d^9 (bc - ad)}{b^{11} (a + bx)} + \frac{d^{10} \text{Log}[a + bx]}{b^{11}}
 \end{aligned}$$

Result (type 3, 591 leaves):

$$\begin{aligned}
 & - \frac{1}{2520 b^{11} (a + bx)^{10}} (bc - ad) \\
 & \quad (7381 a^9 d^9 + a^8 b d^8 (4861 c + 71290 d x) + a^7 b^2 d^7 (3601 c^2 + 46090 c d x + 308205 d^2 x^2) + \\
 & \quad a^6 b^3 d^6 (2761 c^3 + 33490 c^2 d x + 194805 c d^2 x^2 + 784080 d^3 x^3) + \\
 & \quad a^5 b^4 d^5 (2131 c^4 + 25090 c^3 d x + 138105 c^2 d^2 x^2 + 481680 c d^3 x^3 + 1296540 d^4 x^4) + a^4 b^5 d^4 \\
 & \quad (1627 c^5 + 18790 c^4 d x + 100305 c^3 d^2 x^2 + 330480 c^2 d^3 x^3 + 767340 c d^4 x^4 + 1450008 d^5 x^5) + \\
 & \quad a^3 b^6 d^3 (1207 c^6 + 13750 c^5 d x + 71955 c^4 d^2 x^2 + 229680 c^3 d^3 x^3 + 502740 c^2 d^4 x^4 + \\
 & \quad 814968 c d^5 x^5 + 1102500 d^6 x^6) + a^2 b^7 d^2 (847 c^7 + 9550 c^6 d x + 49275 c^5 d^2 x^2 + \\
 & \quad 154080 c^4 d^3 x^3 + 326340 c^3 d^4 x^4 + 497448 c^2 d^5 x^5 + 573300 c d^6 x^6 + 554400 d^7 x^7) + \\
 & \quad a b^8 d (532 c^8 + 5950 c^7 d x + 30375 c^6 d^2 x^2 + 93600 c^5 d^3 x^3 + 194040 c^4 d^4 x^4 + \\
 & \quad 285768 c^3 d^5 x^5 + 308700 c^2 d^6 x^6 + 252000 c d^7 x^7 + 170100 d^8 x^8) + \\
 & \quad b^9 (252 c^9 + 2800 c^8 d x + 14175 c^7 d^2 x^2 + 43200 c^6 d^3 x^3 + 88200 c^5 d^4 x^4 + 127008 c^4 d^5 x^5 + \\
 & \quad 132300 c^3 d^6 x^6 + 100800 c^2 d^7 x^7 + 56700 c d^8 x^8 + 25200 d^9 x^9) + \frac{d^{10} \text{Log}[a + bx]}{b^{11}}
 \end{aligned}$$

**Problem 1323: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + dx)^{10}}{(a + bx)^{12}} dx$$

Optimal (type 1, 28 leaves, 1 step):

$$- \frac{(c + dx)^{11}}{11 (bc - ad) (a + bx)^{11}}$$

Result (type 1, 665 leaves):

$$\begin{aligned}
& - \frac{1}{11 b^{11} (a + b x)^{11}} \left( a^{10} d^{10} + a^9 b d^9 (c + 11 d x) + \right. \\
& a^8 b^2 d^8 (c^2 + 11 c d x + 55 d^2 x^2) + a^7 b^3 d^7 (c^3 + 11 c^2 d x + 55 c d^2 x^2 + 165 d^3 x^3) + \\
& a^6 b^4 d^6 (c^4 + 11 c^3 d x + 55 c^2 d^2 x^2 + 165 c d^3 x^3 + 330 d^4 x^4) + \\
& a^5 b^5 d^5 (c^5 + 11 c^4 d x + 55 c^3 d^2 x^2 + 165 c^2 d^3 x^3 + 330 c d^4 x^4 + 462 d^5 x^5) + \\
& a^4 b^6 d^4 (c^6 + 11 c^5 d x + 55 c^4 d^2 x^2 + 165 c^3 d^3 x^3 + 330 c^2 d^4 x^4 + 462 c d^5 x^5 + 462 d^6 x^6) + a^3 b^7 d^3 \\
& (c^7 + 11 c^6 d x + 55 c^5 d^2 x^2 + 165 c^4 d^3 x^3 + 330 c^3 d^4 x^4 + 462 c^2 d^5 x^5 + 462 c d^6 x^6 + 330 d^7 x^7) + \\
& a^2 b^8 d^2 (c^8 + 11 c^7 d x + 55 c^6 d^2 x^2 + 165 c^5 d^3 x^3 + 330 c^4 d^4 x^4 + 462 c^3 d^5 x^5 + 462 c^2 d^6 x^6 + \\
& 330 c d^7 x^7 + 165 d^8 x^8) + a b^9 d (c^9 + 11 c^8 d x + 55 c^7 d^2 x^2 + 165 c^6 d^3 x^3 + \\
& 330 c^5 d^4 x^4 + 462 c^4 d^5 x^5 + 462 c^3 d^6 x^6 + 330 c^2 d^7 x^7 + 165 c d^8 x^8 + 55 d^9 x^9) + \\
& b^{10} (c^{10} + 11 c^9 d x + 55 c^8 d^2 x^2 + 165 c^7 d^3 x^3 + 330 c^6 d^4 x^4 + 462 c^5 d^5 x^5 + \\
& 462 c^4 d^6 x^6 + 330 c^3 d^7 x^7 + 165 c^2 d^8 x^8 + 55 c d^9 x^9 + 11 d^{10} x^{10}) \left. \right)
\end{aligned}$$

**Problem 1324: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^{10}}{(a + b x)^{13}} dx$$

Optimal (type 1, 58 leaves, 2 steps):

$$- \frac{(c + d x)^{11}}{12 (b c - a d) (a + b x)^{12}} + \frac{d (c + d x)^{11}}{132 (b c - a d)^2 (a + b x)^{11}}$$

Result (type 1, 684 leaves):

$$\begin{aligned}
& - \frac{1}{132 b^{11} (a + b x)^{12}} \left( a^{10} d^{10} + 2 a^9 b d^9 (c + 6 d x) + \right. \\
& 3 a^8 b^2 d^8 (c^2 + 8 c d x + 22 d^2 x^2) + 4 a^7 b^3 d^7 (c^3 + 9 c^2 d x + 33 c d^2 x^2 + 55 d^3 x^3) + \\
& a^6 b^4 d^6 (5 c^4 + 48 c^3 d x + 198 c^2 d^2 x^2 + 440 c d^3 x^3 + 495 d^4 x^4) + \\
& 6 a^5 b^5 d^5 (c^5 + 10 c^4 d x + 44 c^3 d^2 x^2 + 110 c^2 d^3 x^3 + 165 c d^4 x^4 + 132 d^5 x^5) + \\
& a^4 b^6 d^4 (7 c^6 + 72 c^5 d x + 330 c^4 d^2 x^2 + 880 c^3 d^3 x^3 + 1485 c^2 d^4 x^4 + 1584 c d^5 x^5 + 924 d^6 x^6) + 4 a^3 b^7 \\
& d^3 (2 c^7 + 21 c^6 d x + 99 c^5 d^2 x^2 + 275 c^4 d^3 x^3 + 495 c^3 d^4 x^4 + 594 c^2 d^5 x^5 + 462 c d^6 x^6 + 198 d^7 x^7) + \\
& 3 a^2 b^8 d^2 (3 c^8 + 32 c^7 d x + 154 c^6 d^2 x^2 + 440 c^5 d^3 x^3 + 825 c^4 d^4 x^4 + 1056 c^3 d^5 x^5 + \\
& 924 c^2 d^6 x^6 + 528 c d^7 x^7 + 165 d^8 x^8) + 2 a b^9 d (5 c^9 + 54 c^8 d x + 264 c^7 d^2 x^2 + 770 c^6 d^3 x^3 + \\
& 1485 c^5 d^4 x^4 + 1980 c^4 d^5 x^5 + 1848 c^3 d^6 x^6 + 1188 c^2 d^7 x^7 + 495 c d^8 x^8 + 110 d^9 x^9) + \\
& b^{10} (11 c^{10} + 120 c^9 d x + 594 c^8 d^2 x^2 + 1760 c^7 d^3 x^3 + 3465 c^6 d^4 x^4 + 4752 c^5 d^5 x^5 + \\
& 4620 c^4 d^6 x^6 + 3168 c^3 d^7 x^7 + 1485 c^2 d^8 x^8 + 440 c d^9 x^9 + 66 d^{10} x^{10}) \left. \right)
\end{aligned}$$

**Problem 1325: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^{10}}{(a + b x)^{14}} dx$$

Optimal (type 1, 89 leaves, 3 steps):

$$- \frac{(c + d x)^{11}}{13 (b c - a d) (a + b x)^{13}} + \frac{d (c + d x)^{11}}{78 (b c - a d)^2 (a + b x)^{12}} - \frac{d^2 (c + d x)^{11}}{858 (b c - a d)^3 (a + b x)^{11}}$$

Result (type 1, 690 leaves):

$$\begin{aligned}
 & - \frac{1}{858 b^{11} (a + b x)^{13}} (a^{10} d^{10} + a^9 b d^9 (3 c + 13 d x) + \\
 & 3 a^8 b^2 d^8 (2 c^2 + 13 c d x + 26 d^2 x^2) + 2 a^7 b^3 d^7 (5 c^3 + 39 c^2 d x + 117 c d^2 x^2 + 143 d^3 x^3) + \\
 & a^6 b^4 d^6 (15 c^4 + 130 c^3 d x + 468 c^2 d^2 x^2 + 858 c d^3 x^3 + 715 d^4 x^4) + \\
 & 3 a^5 b^5 d^5 (7 c^5 + 65 c^4 d x + 260 c^3 d^2 x^2 + 572 c^2 d^3 x^3 + 715 c d^4 x^4 + 429 d^5 x^5) + \\
 & a^4 b^6 d^4 (28 c^6 + 273 c^5 d x + 1170 c^4 d^2 x^2 + 2860 c^3 d^3 x^3 + 4290 c^2 d^4 x^4 + 3861 c d^5 x^5 + 1716 d^6 x^6) + \\
 & 2 a^3 b^7 d^3 (18 c^7 + 182 c^6 d x + 819 c^5 d^2 x^2 + 2145 c^4 d^3 x^3 + 3575 c^3 d^4 x^4 + \\
 & 3861 c^2 d^5 x^5 + 2574 c d^6 x^6 + 858 d^7 x^7) + 3 a^2 b^8 d^2 (15 c^8 + 156 c^7 d x + 728 c^6 d^2 x^2 + \\
 & 2002 c^5 d^3 x^3 + 3575 c^4 d^4 x^4 + 4290 c^3 d^5 x^5 + 3432 c^2 d^6 x^6 + 1716 c d^7 x^7 + 429 d^8 x^8) + \\
 & a b^9 d (55 c^9 + 585 c^8 d x + 2808 c^7 d^2 x^2 + 8008 c^6 d^3 x^3 + 15015 c^5 d^4 x^4 + 19305 c^4 d^5 x^5 + \\
 & 17160 c^3 d^6 x^6 + 10296 c^2 d^7 x^7 + 3861 c d^8 x^8 + 715 d^9 x^9) + \\
 & b^{10} (66 c^{10} + 715 c^9 d x + 3510 c^8 d^2 x^2 + 10296 c^7 d^3 x^3 + 20020 c^6 d^4 x^4 + 27027 c^5 d^5 x^5 + \\
 & 25740 c^4 d^6 x^6 + 17160 c^3 d^7 x^7 + 7722 c^2 d^8 x^8 + 2145 c d^9 x^9 + 286 d^{10} x^{10})
 \end{aligned}$$

Problem 1326: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^{15}} dx$$

Optimal (type 1, 120 leaves, 4 steps):

$$\begin{aligned}
 & - \frac{(c + d x)^{11}}{14 (b c - a d) (a + b x)^{14}} + \frac{3 d (c + d x)^{11}}{182 (b c - a d)^2 (a + b x)^{13}} - \\
 & \frac{d^2 (c + d x)^{11}}{364 (b c - a d)^3 (a + b x)^{12}} + \frac{d^3 (c + d x)^{11}}{4004 (b c - a d)^4 (a + b x)^{11}}
 \end{aligned}$$

Result (type 1, 692 leaves):

$$\begin{aligned}
 & - \frac{1}{4004 b^{11} (a + b x)^{14}} (a^{10} d^{10} + 2 a^9 b d^9 (2 c + 7 d x) + \\
 & a^8 b^2 d^8 (10 c^2 + 56 c d x + 91 d^2 x^2) + 4 a^7 b^3 d^7 (5 c^3 + 35 c^2 d x + 91 c d^2 x^2 + 91 d^3 x^3) + \\
 & 7 a^6 b^4 d^6 (5 c^4 + 40 c^3 d x + 130 c^2 d^2 x^2 + 208 c d^3 x^3 + 143 d^4 x^4) + \\
 & 14 a^5 b^5 d^5 (4 c^5 + 35 c^4 d x + 130 c^3 d^2 x^2 + 260 c^2 d^3 x^3 + 286 c d^4 x^4 + 143 d^5 x^5) + \\
 & 7 a^4 b^6 d^4 (12 c^6 + 112 c^5 d x + 455 c^4 d^2 x^2 + 1040 c^3 d^3 x^3 + 1430 c^2 d^4 x^4 + 1144 c d^5 x^5 + 429 d^6 x^6) + \\
 & 4 a^3 b^7 d^3 (30 c^7 + 294 c^6 d x + 1274 c^5 d^2 x^2 + 3185 c^4 d^3 x^3 + 5005 c^3 d^4 x^4 + 5005 c^2 d^5 x^5 + \\
 & 3003 c d^6 x^6 + 858 d^7 x^7) + a^2 b^8 d^2 (165 c^8 + 1680 c^7 d x + 7644 c^6 d^2 x^2 + 20384 c^5 d^3 x^3 + \\
 & 35035 c^4 d^4 x^4 + 40040 c^3 d^5 x^5 + 30030 c^2 d^6 x^6 + 13728 c d^7 x^7 + 3003 d^8 x^8) + \\
 & 2 a b^9 d (110 c^9 + 1155 c^8 d x + 5460 c^7 d^2 x^2 + 15288 c^6 d^3 x^3 + 28028 c^5 d^4 x^4 + \\
 & 35035 c^4 d^5 x^5 + 30030 c^3 d^6 x^6 + 17160 c^2 d^7 x^7 + 6006 c d^8 x^8 + 1001 d^9 x^9) + \\
 & b^{10} (286 c^{10} + 3080 c^9 d x + 15015 c^8 d^2 x^2 + 43680 c^7 d^3 x^3 + 84084 c^6 d^4 x^4 + 112112 c^5 d^5 x^5 + \\
 & 105105 c^4 d^6 x^6 + 68640 c^3 d^7 x^7 + 30030 c^2 d^8 x^8 + 8008 c d^9 x^9 + 1001 d^{10} x^{10})
 \end{aligned}$$

### Problem 1327: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+dx)^{10}}{(a+bx)^{16}} dx$$

Optimal (type 1, 151 leaves, 5 steps):

$$-\frac{(c+dx)^{11}}{15(bc-ad)(a+bx)^{15}} + \frac{2d(c+dx)^{11}}{105(bc-ad)^2(a+bx)^{14}} - \frac{2d^2(c+dx)^{11}}{455(bc-ad)^3(a+bx)^{13}} + \frac{d^3(c+dx)^{11}}{1365(bc-ad)^4(a+bx)^{12}} - \frac{d^4(c+dx)^{11}}{15015(bc-ad)^5(a+bx)^{11}}$$

Result (type 1, 690 leaves):

$$-\frac{1}{15015b^{11}(a+bx)^{15}} (a^{10}d^{10} + 5a^9bd^9(c+3dx) + 15a^8b^2d^8(c^2+5cdx+7d^2x^2) + 5a^7b^3d^7(7c^3+45c^2dx+105cd^2x^2+91d^3x^3) + 35a^6b^4d^6(2c^4+15c^3dx+45c^2d^2x^2+65cd^3x^3+39d^4x^4) + 21a^5b^5d^5(6c^5+50c^4dx+175c^3d^2x^2+325c^2d^3x^3+325cd^4x^4+143d^5x^5) + 35a^4b^6d^4(6c^6+54c^5dx+210c^4d^2x^2+455c^3d^3x^3+585c^2d^4x^4+429cd^5x^5+143d^6x^6) + 5a^3b^7d^3(66c^7+630c^6dx+2646c^5d^2x^2+6370c^4d^3x^3+9555c^3d^4x^4+9009c^2d^5x^5+5005cd^6x^6+1287d^7x^7) + 15a^2b^8d^2(33c^8+330c^7dx+1470c^6d^2x^2+3822c^5d^3x^3+6370c^4d^4x^4+7007c^3d^5x^5+5005c^2d^6x^6+2145cd^7x^7+429d^8x^8) + 5ab^9d(143c^9+1485c^8dx+6930c^7d^2x^2+19110c^6d^3x^3+34398c^5d^4x^4+42042c^4d^5x^5+35035c^3d^6x^6+19305c^2d^7x^7+6435cd^8x^8+1001d^9x^9) + b^{10}(1001c^{10}+10725c^9dx+51975c^8d^2x^2+150150c^7d^3x^3+286650c^6d^4x^4+378378c^5d^5x^5+350350c^4d^6x^6+225225c^3d^7x^7+96525c^2d^8x^8+25025cd^9x^9+3003d^{10}x^{10}))$$

### Problem 1328: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+dx)^{10}}{(a+bx)^{17}} dx$$

Optimal (type 1, 182 leaves, 6 steps):

$$-\frac{(c+dx)^{11}}{16(bc-ad)(a+bx)^{16}} + \frac{d(c+dx)^{11}}{48(bc-ad)^2(a+bx)^{15}} - \frac{d^2(c+dx)^{11}}{168(bc-ad)^3(a+bx)^{14}} + \frac{d^3(c+dx)^{11}}{728(bc-ad)^4(a+bx)^{13}} - \frac{d^4(c+dx)^{11}}{4368(bc-ad)^5(a+bx)^{12}} + \frac{d^5(c+dx)^{11}}{48048(bc-ad)^6(a+bx)^{11}}$$

Result (type 1, 694 leaves):



$$\begin{aligned}
 & - \frac{1}{48\,048\,b^{11} (a+b x)^{16}} \left( a^{10} d^{10} + 2 a^9 b d^9 (3 c + 8 d x) + \right. \\
 & \quad 3 a^8 b^2 d^8 (7 c^2 + 32 c d x + 40 d^2 x^2) + 8 a^7 b^3 d^7 (7 c^3 + 42 c^2 d x + 90 c d^2 x^2 + 70 d^3 x^3) + \\
 & \quad 14 a^6 b^4 d^6 (9 c^4 + 64 c^3 d x + 180 c^2 d^2 x^2 + 240 c d^3 x^3 + 130 d^4 x^4) + \\
 & \quad 84 a^5 b^5 d^5 (3 c^5 + 24 c^4 d x + 80 c^3 d^2 x^2 + 140 c^2 d^3 x^3 + 130 c d^4 x^4 + 52 d^5 x^5) + 14 a^4 b^6 d^4 \\
 & \quad \left( 33 c^6 + 288 c^5 d x + 1080 c^4 d^2 x^2 + 2240 c^3 d^3 x^3 + 2730 c^2 d^4 x^4 + 1872 c d^5 x^5 + 572 d^6 x^6 \right) + \\
 & \quad 8 a^3 b^7 d^3 (99 c^7 + 924 c^6 d x + 3780 c^5 d^2 x^2 + 8820 c^4 d^3 x^3 + 12\,740 c^3 d^4 x^4 + 11\,466 c^2 d^5 x^5 + \\
 & \quad 6006 c d^6 x^6 + 1430 d^7 x^7) + 3 a^2 b^8 d^2 (429 c^8 + 4224 c^7 d x + 18\,480 c^6 d^2 x^2 + 47\,040 c^5 d^3 x^3 + \\
 & \quad 76\,440 c^4 d^4 x^4 + 81\,536 c^3 d^5 x^5 + 56\,056 c^2 d^6 x^6 + 22\,880 c d^7 x^7 + 4290 d^8 x^8) + \\
 & \quad 2 a b^9 d (1001 c^9 + 10\,296 c^8 d x + 47\,520 c^7 d^2 x^2 + 129\,360 c^6 d^3 x^3 + 229\,320 c^5 d^4 x^4 + \\
 & \quad 275\,184 c^4 d^5 x^5 + 224\,224 c^3 d^6 x^6 + 120\,120 c^2 d^7 x^7 + 38\,610 c d^8 x^8 + 5720 d^9 x^9) + \\
 & \quad \left. b^{10} (3003 c^{10} + 32\,032 c^9 d x + 154\,440 c^8 d^2 x^2 + 443\,520 c^7 d^3 x^3 + 840\,840 c^6 d^4 x^4 + 1\,100\,736 c^5 d^5 x^5 + \right. \\
 & \quad \left. 1\,009\,008 c^4 d^6 x^6 + 640\,640 c^3 d^7 x^7 + 270\,270 c^2 d^8 x^8 + 68\,640 c d^9 x^9 + 8008 d^{10} x^{10}) \right)
 \end{aligned}$$

Problem 1329: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+d x)^{10}}{(a+b x)^{18}} dx$$

Optimal (type 1, 213 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{(c+d x)^{11}}{17 (b c - a d) (a+b x)^{17}} + \frac{3 d (c+d x)^{11}}{136 (b c - a d)^2 (a+b x)^{16}} - \\
 & \frac{d^2 (c+d x)^{11}}{136 (b c - a d)^3 (a+b x)^{15}} + \frac{d^3 (c+d x)^{11}}{476 (b c - a d)^4 (a+b x)^{14}} - \frac{3 d^4 (c+d x)^{11}}{6188 (b c - a d)^5 (a+b x)^{13}} + \\
 & \frac{d^5 (c+d x)^{11}}{12376 (b c - a d)^6 (a+b x)^{12}} - \frac{d^6 (c+d x)^{11}}{136136 (b c - a d)^7 (a+b x)^{11}}
 \end{aligned}$$

Result (type 1, 690 leaves):

$$\begin{aligned}
 & - \frac{1}{136\,136\,b^{11} (a+b x)^{17}} \left( a^{10} d^{10} + a^9 b d^9 (7 c + 17 d x) + \right. \\
 & \quad a^8 b^2 d^8 (28 c^2 + 119 c d x + 136 d^2 x^2) + 4 a^7 b^3 d^7 (21 c^3 + 119 c^2 d x + 238 c d^2 x^2 + 170 d^3 x^3) + \\
 & \quad 14 a^6 b^4 d^6 (15 c^4 + 102 c^3 d x + 272 c^2 d^2 x^2 + 340 c d^3 x^3 + 170 d^4 x^4) + \\
 & \quad 14 a^5 b^5 d^5 (33 c^5 + 255 c^4 d x + 816 c^3 d^2 x^2 + 1360 c^2 d^3 x^3 + 1190 c d^4 x^4 + 442 d^5 x^5) + 14 a^4 b^6 d^4 \\
 & \quad d^4 (66 c^6 + 561 c^5 d x + 2040 c^4 d^2 x^2 + 4080 c^3 d^3 x^3 + 4760 c^2 d^4 x^4 + 3094 c d^5 x^5 + 884 d^6 x^6) + \\
 & \quad 4 a^3 b^7 d^3 (429 c^7 + 3927 c^6 d x + 15\,708 c^5 d^2 x^2 + 35\,700 c^4 d^3 x^3 + \\
 & \quad 49\,980 c^3 d^4 x^4 + 43\,316 c^2 d^5 x^5 + 21\,658 c d^6 x^6 + 4862 d^7 x^7) + \\
 & \quad a^2 b^8 d^2 (3003 c^8 + 29\,172 c^7 d x + 125\,664 c^6 d^2 x^2 + 314\,160 c^5 d^3 x^3 + 499\,800 c^4 d^4 x^4 + \\
 & \quad 519\,792 c^3 d^5 x^5 + 346\,528 c^2 d^6 x^6 + 136\,136 c d^7 x^7 + 24\,310 d^8 x^8) + \\
 & \quad a b^9 d (5005 c^9 + 51\,051 c^8 d x + 233\,376 c^7 d^2 x^2 + 628\,320 c^6 d^3 x^3 + 1\,099\,560 c^5 d^4 x^4 + \\
 & \quad 1\,299\,480 c^4 d^5 x^5 + 1\,039\,584 c^3 d^6 x^6 + 544\,544 c^2 d^7 x^7 + 170\,170 c d^8 x^8 + 24\,310 d^9 x^9) + \\
 & \quad \left. b^{10} (8008 c^{10} + 85\,085 c^9 d x + 408\,408 c^8 d^2 x^2 + 1\,166\,880 c^7 d^3 x^3 + 2\,199\,120 c^6 d^4 x^4 + 2\,858\,856 c^5 \right. \\
 & \quad \left. d^5 x^5 + 2\,598\,960 c^4 d^6 x^6 + 1\,633\,632 c^3 d^7 x^7 + 680\,680 c^2 d^8 x^8 + 170\,170 c d^9 x^9 + 19\,448 d^{10} x^{10}) \right)
 \end{aligned}$$

### Problem 1330: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+dx)^{10}}{(a+bx)^{19}} dx$$

Optimal (type 1, 244 leaves, 8 steps):

$$\begin{aligned} & -\frac{(c+dx)^{11}}{18(bc-ad)(a+bx)^{18}} + \frac{7d(c+dx)^{11}}{306(bc-ad)^2(a+bx)^{17}} - \\ & \frac{7d^2(c+dx)^{11}}{816(bc-ad)^3(a+bx)^{16}} + \frac{7d^3(c+dx)^{11}}{2448(bc-ad)^4(a+bx)^{15}} - \frac{d^4(c+dx)^{11}}{1224(bc-ad)^5(a+bx)^{14}} + \\ & \frac{d^5(c+dx)^{11}}{5304(bc-ad)^6(a+bx)^{13}} - \frac{d^6(c+dx)^{11}}{31824(bc-ad)^7(a+bx)^{12}} + \frac{d^7(c+dx)^{11}}{350064(bc-ad)^8(a+bx)^{11}} \end{aligned}$$

Result (type 1, 694 leaves):

$$\begin{aligned} & -\frac{1}{350064b^{11}(a+bx)^{18}} (a^{10}d^{10} + 2a^9bd^9(4c+9dx) + \\ & 9a^8b^2d^8(4c^2+16cdx+17d^2x^2) + 24a^7b^3d^7(5c^3+27c^2dx+51cd^2x^2+34d^3x^3) + \\ & 6a^6b^4d^6(55c^4+360c^3dx+918c^2d^2x^2+1088cd^3x^3+510d^4x^4) + \\ & 36a^5b^5d^5(22c^5+165c^4dx+510c^3d^2x^2+816c^2d^3x^3+680cd^4x^4+238d^5x^5) + 6a^4b^6d^4 \\ & (286c^6+2376c^5dx+8415c^4d^2x^2+16320c^3d^3x^3+18360c^2d^4x^4+11424cd^5x^5+3094d^6x^6) + \\ & 24a^3b^7d^3(143c^7+1287c^6dx+5049c^5d^2x^2+11220c^4d^3x^3+15300c^3d^4x^4+12852c^2d^5x^5+ \\ & 6188cd^6x^6+1326d^7x^7) + 9a^2b^8d^2(715c^8+6864c^7dx+29172c^6d^2x^2+71808c^5d^3x^3+ \\ & 112200c^4d^4x^4+114240c^3d^5x^5+74256c^2d^6x^6+28288cd^7x^7+4862d^8x^8) + \\ & 2abd(5720c^9+57915c^8dx+262548c^7d^2x^2+700128c^6d^3x^3+1211760c^5d^4x^4+ \\ & 1413720c^4d^5x^5+1113840c^3d^6x^6+572832c^2d^7x^7+175032cd^8x^8+24310d^9x^9) + b^{10} \\ & (19448c^{10}+205920c^9dx+984555c^8d^2x^2+2800512c^7d^3x^3+5250960c^6d^4x^4+6785856c^5d^5 \\ & x^5+6126120c^4d^6x^6+3818880c^3d^7x^7+1575288c^2d^8x^8+388960cd^9x^9+43758d^{10}x^{10})) \end{aligned}$$

### Problem 1331: Result more than twice size of optimal antiderivative.

$$\int \frac{(c+dx)^{10}}{(a+bx)^{20}} dx$$

Optimal (type 1, 273 leaves, 2 steps):

$$\begin{aligned} & -\frac{(bc-ad)^{10}}{19b^{11}(a+bx)^{19}} - \frac{5d(bc-ad)^9}{9b^{11}(a+bx)^{18}} - \frac{45d^2(bc-ad)^8}{17b^{11}(a+bx)^{17}} - \\ & \frac{15d^3(bc-ad)^7}{2b^{11}(a+bx)^{16}} - \frac{14d^4(bc-ad)^6}{b^{11}(a+bx)^{15}} - \frac{18d^5(bc-ad)^5}{b^{11}(a+bx)^{14}} - \frac{210d^6(bc-ad)^4}{13b^{11}(a+bx)^{13}} - \\ & \frac{10d^7(bc-ad)^3}{b^{11}(a+bx)^{12}} - \frac{45d^8(bc-ad)^2}{11b^{11}(a+bx)^{11}} - \frac{d^9(bc-ad)}{b^{11}(a+bx)^{10}} - \frac{d^{10}}{9b^{11}(a+bx)^9} \end{aligned}$$

Result (type 1, 692 leaves):

$$\begin{aligned}
 & - \frac{1}{831402 b^{11} (a + b x)^{19}} \left( a^{10} d^{10} + a^9 b d^9 (9 c + 19 d x) + \right. \\
 & 9 a^8 b^2 d^8 (5 c^2 + 19 c d x + 19 d^2 x^2) + 3 a^7 b^3 d^7 (55 c^3 + 285 c^2 d x + 513 c d^2 x^2 + 323 d^3 x^3) + \\
 & 3 a^6 b^4 d^6 (165 c^4 + 1045 c^3 d x + 2565 c^2 d^2 x^2 + 2907 c d^3 x^3 + 1292 d^4 x^4) + \\
 & 9 a^5 b^5 d^5 (143 c^5 + 1045 c^4 d x + 3135 c^3 d^2 x^2 + 4845 c^2 d^3 x^3 + 3876 c d^4 x^4 + 1292 d^5 x^5) + \\
 & 3 a^4 b^6 d^4 (1001 c^6 + 8151 c^5 d x + 28215 c^4 d^2 x^2 + 53295 c^3 d^3 x^3 + 58140 c^2 d^4 x^4 + \\
 & 34884 c d^5 x^5 + 9044 d^6 x^6) + 3 a^3 b^7 d^3 (2145 c^7 + 19019 c^6 d x + 73359 c^5 d^2 x^2 + \\
 & 159885 c^4 d^3 x^3 + 213180 c^3 d^4 x^4 + 174420 c^2 d^5 x^5 + 81396 c d^6 x^6 + 16796 d^7 x^7) + \\
 & 9 a^2 b^8 d^2 (1430 c^8 + 13585 c^7 d x + 57057 c^6 d^2 x^2 + 138567 c^5 d^3 x^3 + 213180 c^4 d^4 x^4 + \\
 & 213180 c^3 d^5 x^5 + 135660 c^2 d^6 x^6 + 50388 c d^7 x^7 + 8398 d^8 x^8) + \\
 & a b^9 d (24310 c^9 + 244530 c^8 d x + 1100385 c^7 d^2 x^2 + 2909907 c^6 d^3 x^3 + 4988412 c^5 d^4 x^4 + \\
 & 5755860 c^4 d^5 x^5 + 4476780 c^3 d^6 x^6 + 2267460 c^2 d^7 x^7 + 680238 c d^8 x^8 + 92378 d^9 x^9) + \\
 & b^{10} (43758 c^{10} + 461890 c^9 d x + 2200770 c^8 d^2 x^2 + 6235515 c^7 d^3 x^3 + \\
 & 11639628 c^6 d^4 x^4 + 14965236 c^5 d^5 x^5 + 13430340 c^4 d^6 x^6 + \\
 & 8314020 c^3 d^7 x^7 + 3401190 c^2 d^8 x^8 + 831402 c d^9 x^9 + 92378 d^{10} x^{10}) \left. \right)
 \end{aligned}$$

Problem 1332: Result more than twice size of optimal antiderivative.

$$\int \frac{(c + d x)^{10}}{(a + b x)^{21}} dx$$

Optimal (type 1, 279 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{(b c - a d)^{10}}{20 b^{11} (a + b x)^{20}} - \frac{10 d (b c - a d)^9}{19 b^{11} (a + b x)^{19}} - \frac{5 d^2 (b c - a d)^8}{2 b^{11} (a + b x)^{18}} - \\
 & \frac{120 d^3 (b c - a d)^7}{17 b^{11} (a + b x)^{17}} - \frac{105 d^4 (b c - a d)^6}{8 b^{11} (a + b x)^{16}} - \frac{84 d^5 (b c - a d)^5}{5 b^{11} (a + b x)^{15}} - \frac{15 d^6 (b c - a d)^4}{b^{11} (a + b x)^{14}} - \\
 & \frac{120 d^7 (b c - a d)^3}{13 b^{11} (a + b x)^{13}} - \frac{15 d^8 (b c - a d)^2}{4 b^{11} (a + b x)^{12}} - \frac{10 d^9 (b c - a d)}{11 b^{11} (a + b x)^{11}} - \frac{d^{10}}{10 b^{11} (a + b x)^{10}}
 \end{aligned}$$

Result (type 1, 692 leaves):

$$\begin{aligned}
 & - \frac{1}{1847560 b^{11} (a + b x)^{20}} \left( a^{10} d^{10} + 10 a^9 b d^9 (c + 2 d x) + \right. \\
 & 5 a^8 b^2 d^8 (11 c^2 + 40 c d x + 38 d^2 x^2) + 20 a^7 b^3 d^7 (11 c^3 + 55 c^2 d x + 95 c d^2 x^2 + 57 d^3 x^3) + \\
 & 5 a^6 b^4 d^6 (143 c^4 + 880 c^3 d x + 2090 c^2 d^2 x^2 + 2280 c d^3 x^3 + 969 d^4 x^4) + \\
 & 2 a^5 b^5 d^5 (1001 c^5 + 7150 c^4 d x + 20900 c^3 d^2 x^2 + 31350 c^2 d^3 x^3 + 24225 c d^4 x^4 + 7752 d^5 x^5) + \\
 & 5 a^4 b^6 d^4 (1001 c^6 + 8008 c^5 d x + 27170 c^4 d^2 x^2 + 50160 c^3 d^3 x^3 + 53295 c^2 d^4 x^4 + \\
 & 31008 c d^5 x^5 + 7752 d^6 x^6) + 20 a^3 b^7 d^3 (572 c^7 + 5005 c^6 d x + 19019 c^5 d^2 x^2 + \\
 & 40755 c^4 d^3 x^3 + 53295 c^3 d^4 x^4 + 42636 c^2 d^5 x^5 + 19380 c d^6 x^6 + 3876 d^7 x^7) + \\
 & 5 a^2 b^8 d^2 (4862 c^8 + 45760 c^7 d x + 190190 c^6 d^2 x^2 + 456456 c^5 d^3 x^3 + 692835 c^4 d^4 x^4 + \\
 & 682176 c^3 d^5 x^5 + 426360 c^2 d^6 x^6 + 155040 c d^7 x^7 + 25194 d^8 x^8) + \\
 & 10 a b^9 d (4862 c^9 + 48620 c^8 d x + 217360 c^7 d^2 x^2 + 570570 c^6 d^3 x^3 + 969969 c^5 d^4 x^4 + \\
 & 1108536 c^4 d^5 x^5 + 852720 c^3 d^6 x^6 + 426360 c^2 d^7 x^7 + 125970 c d^8 x^8 + 16796 d^9 x^9) + \\
 & b^{10} (92378 c^{10} + 972400 c^9 d x + 4618900 c^8 d^2 x^2 + 13041600 c^7 d^3 x^3 + \\
 & 24249225 c^6 d^4 x^4 + 31039008 c^5 d^5 x^5 + 27713400 c^4 d^6 x^6 + \\
 & 17054400 c^3 d^7 x^7 + 6928350 c^2 d^8 x^8 + 1679600 c d^9 x^9 + 184756 d^{10} x^{10}) \left. \right)
 \end{aligned}$$

**Problem 1333: Result more than twice size of optimal antiderivative.**

$$\int \frac{(c + d x)^{10}}{(a + b x)^{22}} dx$$

Optimal (type 1, 279 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{(b c - a d)^{10}}{21 b^{11} (a + b x)^{21}} - \frac{d (b c - a d)^9}{2 b^{11} (a + b x)^{20}} - \frac{45 d^2 (b c - a d)^8}{19 b^{11} (a + b x)^{19}} - \\
 & \frac{20 d^3 (b c - a d)^7}{3 b^{11} (a + b x)^{18}} - \frac{210 d^4 (b c - a d)^6}{17 b^{11} (a + b x)^{17}} - \frac{63 d^5 (b c - a d)^5}{4 b^{11} (a + b x)^{16}} - \frac{14 d^6 (b c - a d)^4}{b^{11} (a + b x)^{15}} - \\
 & \frac{60 d^7 (b c - a d)^3}{7 b^{11} (a + b x)^{14}} - \frac{45 d^8 (b c - a d)^2}{13 b^{11} (a + b x)^{13}} - \frac{5 d^9 (b c - a d)}{6 b^{11} (a + b x)^{12}} - \frac{d^{10}}{11 b^{11} (a + b x)^{11}}
 \end{aligned}$$

Result (type 1, 692 leaves):

$$\begin{aligned}
 & - \frac{1}{3879876 b^{11} (a + b x)^{21}} \left( a^{10} d^{10} + a^9 b d^9 (11 c + 21 d x) + \right. \\
 & 3 a^8 b^2 d^8 (22 c^2 + 77 c d x + 70 d^2 x^2) + 2 a^7 b^3 d^7 (143 c^3 + 693 c^2 d x + 1155 c d^2 x^2 + 665 d^3 x^3) + \\
 & 7 a^6 b^4 d^6 (143 c^4 + 858 c^3 d x + 1980 c^2 d^2 x^2 + 2090 c d^3 x^3 + 855 d^4 x^4) + \\
 & 21 a^5 b^5 d^5 (143 c^5 + 1001 c^4 d x + 2860 c^3 d^2 x^2 + 4180 c^2 d^3 x^3 + 3135 c d^4 x^4 + 969 d^5 x^5) + \\
 & 7 a^4 b^6 d^4 (1144 c^6 + 9009 c^5 d x + 30030 c^4 d^2 x^2 + 54340 c^3 d^3 x^3 + 56430 c^2 d^4 x^4 + \\
 & 31977 c d^5 x^5 + 7752 d^6 x^6) + 2 a^3 b^7 d^3 (9724 c^7 + 84084 c^6 d x + 315315 c^5 d^2 x^2 + \\
 & 665665 c^4 d^3 x^3 + 855855 c^3 d^4 x^4 + 671517 c^2 d^5 x^5 + 298452 c d^6 x^6 + 58140 d^7 x^7) + \\
 & 3 a^2 b^8 d^2 (14586 c^8 + 136136 c^7 d x + 560560 c^6 d^2 x^2 + 1331330 c^5 d^3 x^3 + 1996995 c^4 d^4 x^4 + \\
 & 1939938 c^3 d^5 x^5 + 1193808 c^2 d^6 x^6 + 426360 c d^7 x^7 + 67830 d^8 x^8) + \\
 & a b^9 d (92378 c^9 + 918918 c^8 d x + 4084080 c^7 d^2 x^2 + 10650640 c^6 d^3 x^3 + 17972955 c^5 d^4 x^4 + \\
 & 20369349 c^4 d^5 x^5 + 15519504 c^3 d^6 x^6 + 7674480 c^2 d^7 x^7 + 2238390 c d^8 x^8 + 293930 d^9 x^9) + \\
 & b^{10} (184756 c^{10} + 1939938 c^9 d x + 9189180 c^8 d^2 x^2 + 25865840 c^7 d^3 x^3 + \\
 & 47927880 c^6 d^4 x^4 + 61108047 c^5 d^5 x^5 + 54318264 c^4 d^6 x^6 + \\
 & 33256080 c^3 d^7 x^7 + 13430340 c^2 d^8 x^8 + 3233230 c d^9 x^9 + 352716 d^{10} x^{10}) \left. \right)
 \end{aligned}$$

**Problem 1362: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^9}{(c + d x)^8} dx$$

Optimal (type 3, 232 leaves, 2 steps):

$$\begin{aligned}
 & - \frac{b^8 (8 b c - 9 a d) x}{d^9} + \frac{b^9 x^2}{2 d^8} + \frac{(b c - a d)^9}{7 d^{10} (c + d x)^7} - \frac{3 b (b c - a d)^8}{2 d^{10} (c + d x)^6} + \frac{36 b^2 (b c - a d)^7}{5 d^{10} (c + d x)^5} - \frac{21 b^3 (b c - a d)^6}{d^{10} (c + d x)^4} \\
 & + \frac{42 b^4 (b c - a d)^5}{d^{10} (c + d x)^3} - \frac{63 b^5 (b c - a d)^4}{d^{10} (c + d x)^2} + \frac{84 b^6 (b c - a d)^3}{d^{10} (c + d x)} + \frac{36 b^7 (b c - a d)^2 \text{Log}[c + d x]}{d^{10}}
 \end{aligned}$$

Result (type 3, 584 leaves):

$$\begin{aligned}
 & - \frac{1}{70 d^{10} (c + d x)^7} \left( 10 a^9 d^9 + 15 a^8 b d^8 (c + 7 d x) + 24 a^7 b^2 d^7 (c^2 + 7 c d x + 21 d^2 x^2) + 42 a^6 b^3 d^6 \right. \\
 & (c^3 + 7 c^2 d x + 21 c d^2 x^2 + 35 d^3 x^3) + 84 a^5 b^4 d^5 (c^4 + 7 c^3 d x + 21 c^2 d^2 x^2 + 35 c d^3 x^3 + 35 d^4 x^4) + \\
 & 210 a^4 b^5 d^4 (c^5 + 7 c^4 d x + 21 c^3 d^2 x^2 + 35 c^2 d^3 x^3 + 35 c d^4 x^4 + 21 d^5 x^5) + \\
 & 840 a^3 b^6 d^3 (c^6 + 7 c^5 d x + 21 c^4 d^2 x^2 + 35 c^3 d^3 x^3 + 35 c^2 d^4 x^4 + 21 c d^5 x^5 + 7 d^6 x^6) - \\
 & 6 a^2 b^7 c d^2 (1089 c^6 + 7203 c^5 d x + 20139 c^4 d^2 x^2 + 30625 c^3 d^3 x^3 + 26950 c^2 d^4 x^4 + \\
 & 13230 c d^5 x^5 + 2940 d^6 x^6) + 6 a b^8 d (1443 c^8 + 9261 c^7 d x + 24843 c^6 d^2 x^2 + \\
 & 35525 c^5 d^3 x^3 + 28175 c^4 d^4 x^4 + 11025 c^3 d^5 x^5 + 735 c^2 d^6 x^6 - 735 c d^7 x^7 - 105 d^8 x^8) - \\
 & b^9 (3349 c^9 + 20923 c^8 d x + 53949 c^7 d^2 x^2 + 72275 c^6 d^3 x^3 + 50225 c^5 d^4 x^4 + 12495 c^4 d^5 x^5 - 4655 \\
 & c^3 d^6 x^6 - 3185 c^2 d^7 x^7 - 315 c d^8 x^8 + 35 d^9 x^9) - 2520 b^7 (b c - a d)^2 (c + d x)^7 \text{Log}[c + d x] \left. \right)
 \end{aligned}$$

**Problem 1363: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + b x)^8}{(c + d x)^8} dx$$

Optimal (type 3, 209 leaves, 2 steps):

$$\frac{b^8 x}{d^8} - \frac{(bc - ad)^8}{7 d^9 (c + dx)^7} + \frac{4 b (bc - ad)^7}{3 d^9 (c + dx)^6} - \frac{28 b^2 (bc - ad)^6}{5 d^9 (c + dx)^5} + \frac{14 b^3 (bc - ad)^5}{d^9 (c + dx)^4} -$$

$$\frac{70 b^4 (bc - ad)^4}{3 d^9 (c + dx)^3} + \frac{28 b^5 (bc - ad)^3}{d^9 (c + dx)^2} - \frac{28 b^6 (bc - ad)^2}{d^9 (c + dx)} - \frac{8 b^7 (bc - ad) \operatorname{Log}[c + dx]}{d^9}$$

Result (type 3, 474 leaves):

$$-\frac{1}{105 d^9 (c + dx)^7} \left( 15 a^8 d^8 + 20 a^7 b d^7 (c + 7 dx) + 28 a^6 b^2 d^6 (c^2 + 7 c dx + 21 d^2 x^2) + 42 a^5 b^3 d^5 \right.$$

$$\left. (c^3 + 7 c^2 dx + 21 c d^2 x^2 + 35 d^3 x^3) + 70 a^4 b^4 d^4 (c^4 + 7 c^3 dx + 21 c^2 d^2 x^2 + 35 c d^3 x^3 + 35 d^4 x^4) + \right.$$

$$\left. 140 a^3 b^5 d^3 (c^5 + 7 c^4 dx + 21 c^3 d^2 x^2 + 35 c^2 d^3 x^3 + 35 c d^4 x^4 + 21 d^5 x^5) + \right.$$

$$\left. 420 a^2 b^6 d^2 (c^6 + 7 c^5 dx + 21 c^4 d^2 x^2 + 35 c^3 d^3 x^3 + 35 c^2 d^4 x^4 + 21 c d^5 x^5 + 7 d^6 x^6) - \right.$$

$$\left. 2 a b^7 c d (1089 c^6 + 7203 c^5 dx + 20139 c^4 d^2 x^2 + \right.$$

$$\left. 30625 c^3 d^3 x^3 + 26950 c^2 d^4 x^4 + 13230 c d^5 x^5 + 2940 d^6 x^6) + \right.$$

$$\left. b^8 (1443 c^8 + 9261 c^7 dx + 24843 c^6 d^2 x^2 + 35525 c^5 d^3 x^3 + 28175 c^4 d^4 x^4 + 11025 c^3 d^5 x^5 + \right.$$

$$\left. 735 c^2 d^6 x^6 - 735 c d^7 x^7 - 105 d^8 x^8) + 840 b^7 (bc - ad) (c + dx)^7 \operatorname{Log}[c + dx] \right)$$

**Problem 1365: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + bx)^6}{(c + dx)^8} dx$$

Optimal (type 1, 28 leaves, 1 step):

$$\frac{(a + bx)^7}{7 (bc - ad) (c + dx)^7}$$

Result (type 1, 271 leaves):

$$-\frac{1}{7 d^7 (c + dx)^7} \left( a^6 d^6 + a^5 b d^5 (c + 7 dx) + a^4 b^2 d^4 (c^2 + 7 c dx + 21 d^2 x^2) + a^3 b^3 d^3 \right.$$

$$\left. (c^3 + 7 c^2 dx + 21 c d^2 x^2 + 35 d^3 x^3) + a^2 b^4 d^2 (c^4 + 7 c^3 dx + 21 c^2 d^2 x^2 + 35 c d^3 x^3 + 35 d^4 x^4) + \right.$$

$$\left. a b^5 d (c^5 + 7 c^4 dx + 21 c^3 d^2 x^2 + 35 c^2 d^3 x^3 + 35 c d^4 x^4 + 21 d^5 x^5) + \right.$$

$$\left. b^6 (c^6 + 7 c^5 dx + 21 c^4 d^2 x^2 + 35 c^3 d^3 x^3 + 35 c^2 d^4 x^4 + 21 c d^5 x^5 + 7 d^6 x^6) \right)$$

**Problem 1366: Result more than twice size of optimal antiderivative.**

$$\int \frac{(a + bx)^5}{(c + dx)^8} dx$$

Optimal (type 1, 58 leaves, 2 steps):

$$\frac{(a + bx)^6}{7 (bc - ad) (c + dx)^7} + \frac{b (a + bx)^6}{42 (bc - ad)^2 (c + dx)^6}$$

Result (type 1, 205 leaves):

$$\begin{aligned}
 & - \frac{1}{42 d^6 (c + d x)^7} (6 a^5 d^5 + 5 a^4 b d^4 (c + 7 d x) + \\
 & 4 a^3 b^2 d^3 (c^2 + 7 c d x + 21 d^2 x^2) + 3 a^2 b^3 d^2 (c^3 + 7 c^2 d x + 21 c d^2 x^2 + 35 d^3 x^3) + \\
 & 2 a b^4 d (c^4 + 7 c^3 d x + 21 c^2 d^2 x^2 + 35 c d^3 x^3 + 35 d^4 x^4) + \\
 & b^5 (c^5 + 7 c^4 d x + 21 c^3 d^2 x^2 + 35 c^2 d^3 x^3 + 35 c d^4 x^4 + 21 d^5 x^5))
 \end{aligned}$$

**Problem 1453: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(-2+x)\sqrt{2+x}} dx$$

Optimal (type 3, 14 leaves, 2 steps):

$$-\text{ArcTanh}\left[\frac{\sqrt{2+x}}{2}\right]$$

Result (type 3, 31 leaves):

$$\frac{1}{2} \text{Log}[2 - \sqrt{2+x}] - \frac{1}{2} \text{Log}[2 + \sqrt{2+x}]$$

**Problem 1458: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)(c+dx)^{1/3}} dx$$

Optimal (type 3, 139 leaves, 4 steps):

$$\frac{\sqrt{3} \text{ArcTan}\left[\frac{1 + \frac{2b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3}}}{\sqrt{3}}\right]}{b^{2/3}(bc-ad)^{1/3}} - \frac{\text{Log}[a+bx]}{2b^{2/3}(bc-ad)^{1/3}} + \frac{3 \text{Log}[(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}]}{2b^{2/3}(bc-ad)^{1/3}}$$

Result (type 5, 47 leaves):

$$- \frac{3(c+dx)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, 1, \frac{5}{3}, \frac{b(c+dx)}{bc-ad}\right]}{2bc - 2ad}$$

**Problem 1459: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)(c+dx)^{2/3}} dx$$

Optimal (type 3, 140 leaves, 4 steps):

$$- \frac{\sqrt{3} \text{ArcTan}\left[\frac{1 + \frac{2b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3}}}{\sqrt{3}}\right]}{b^{1/3}(bc-ad)^{2/3}} - \frac{\text{Log}[a+bx]}{2b^{1/3}(bc-ad)^{2/3}} + \frac{3 \text{Log}[(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}]}{2b^{1/3}(bc-ad)^{2/3}}$$

Result (type 5, 46 leaves):

$$\frac{3 (c + d x)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{b(c+dx)}{bc-ad}\right]}{bc - ad}$$

**Problem 1539: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-bx} \sqrt{2+bx}} dx$$

Optimal (type 3, 10 leaves, 3 steps):

$$\frac{\text{ArcSin}[1 + bx]}{b}$$

Result (type 3, 51 leaves):

$$\frac{2 \sqrt{x} \sqrt{2+bx} \text{ArcSinh}\left[\frac{\sqrt{b}\sqrt{x}}{\sqrt{2}}\right]}{\sqrt{b} \sqrt{-bx(2+bx)}}$$

**Problem 1540: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-1-bx} \sqrt{2+bx}} dx$$

Optimal (type 3, 11 leaves, 3 steps):

$$\frac{\text{ArcSin}[3 + 2bx]}{b}$$

Result (type 3, 49 leaves):

$$\frac{2 \sqrt{1+bx} \sqrt{2+bx} \text{ArcSinh}[\sqrt{1+bx}]}{b \sqrt{-(1+bx)(2+bx)}}$$

**Problem 1550: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{\sqrt{-4+bx} \sqrt{4+bx}} dx$$

Optimal (type 3, 11 leaves, 1 step):

$$\frac{\text{ArcCosh}\left[\frac{bx}{4}\right]}{b}$$

Result (type 3, 24 leaves):

$$\frac{2 \text{ArcSinh}\left[\frac{\sqrt{-4+bx}}{2\sqrt{2}}\right]}{b}$$



Problem 1555: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{4-x} \sqrt{x}} dx$$

Optimal (type 3, 10 leaves, 3 steps):

$$-\text{ArcSin}\left[1 - \frac{x}{2}\right]$$

Result (type 3, 38 leaves):

$$\frac{2 \sqrt{-4+x} \sqrt{x} \text{Log}\left[\sqrt{-4+x} + \sqrt{x}\right]}{\sqrt{-(-4+x)x}}$$

Problem 1558: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{a-bx} \sqrt{c+dx}} dx$$

Optimal (type 3, 43 leaves, 3 steps):

$$\frac{2 \text{ArcTan}\left[\frac{\sqrt{d} \sqrt{a-bx}}{\sqrt{b} \sqrt{c+dx}}\right]}{\sqrt{b} \sqrt{d}}$$

Result (type 3, 64 leaves):

$$\frac{i \text{Log}\left[2 \sqrt{a-bx} \sqrt{c+dx} - \frac{i(bc-ad+2bdx)}{\sqrt{b} \sqrt{d}}\right]}{\sqrt{b} \sqrt{d}}$$

Problem 1559: Result unnecessarily involves higher level functions.

$$\int (a+bx)^{3/2} (c+dx)^{1/3} dx$$

Optimal (type 4, 457 leaves, 5 steps):

$$\begin{aligned}
 & - \frac{108 (bc - ad)^2 \sqrt{a + bx} (c + dx)^{1/3}}{935 b d^2} + \frac{12 (bc - ad) (a + bx)^{3/2} (c + dx)^{1/3}}{187 b d} + \\
 & \frac{6 (a + bx)^{5/2} (c + dx)^{1/3}}{17 b} - \left( 108 \times 3^{3/4} \sqrt{2 - \sqrt{3}} (bc - ad)^3 \left( (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right) \right. \\
 & \sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c + dx)^{1/3} + b^{2/3} (c + dx)^{2/3}}{\left( (1 - \sqrt{3}) (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right)^2}} \\
 & \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 + \sqrt{3}) (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3}}{(1 - \sqrt{3}) (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3}} \right], -7 + 4 \sqrt{3} \right] \right) / \\
 & \left( 935 b^{4/3} d^3 \sqrt{a + bx} \sqrt{-\frac{(bc - ad)^{1/3} \left( (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right)}{\left( (1 - \sqrt{3}) (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right)^2}} \right)
 \end{aligned}$$

Result (type 5, 142 leaves):

$$\begin{aligned}
 & - \frac{1}{935 b d^3 \sqrt{a + bx}} \\
 & 6 (c + dx)^{1/3} \left( -d (a + bx) (27 a^2 d^2 + 2 a b d (23 c + 50 d x) + b^2 (-18 c^2 + 10 c d x + 55 d^2 x^2)) - \right. \\
 & \left. 27 (bc - ad)^3 \sqrt{\frac{d (a + bx)}{-bc + ad}} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b (c + dx)}{bc - ad} \right] \right)
 \end{aligned}$$

**Problem 1560: Result unnecessarily involves higher level functions.**

$$\int \sqrt{a + bx} (c + dx)^{1/3} dx$$

Optimal (type 4, 419 leaves, 4 steps):

$$\frac{12 (b c - a d) \sqrt{a + b x} (c + d x)^{1/3}}{55 b d} + \frac{6 (a + b x)^{3/2} (c + d x)^{1/3}}{11 b} +$$

$$\left( 12 \times 3^{3/4} \sqrt{2 - \sqrt{3}} (b c - a d)^2 \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \right.$$

$$\sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}}$$

$$\text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 + \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{(1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}} \right], -7 + 4 \sqrt{3} \right] \Big/$$

$$\left( 55 b^{4/3} d^2 \sqrt{a + b x} \sqrt{-\frac{(b c - a d)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right)$$

Result (type 5, 110 leaves):

$$\frac{1}{55 b d^2 \sqrt{a + b x}} 6 (c + d x)^{1/3} \left( d (a + b x) (2 b c + 3 a d + 5 b d x) - \right.$$

$$\left. 3 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

**Problem 1561: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + d x)^{1/3}}{\sqrt{a + b x}} dx$$

Optimal (type 4, 381 leaves, 3 steps):

$$\frac{6 \sqrt{a+bx} (c+dx)^{1/3}}{5b} - \left( 4 \times 3^{3/4} \sqrt{2-\sqrt{3}} (bc-ad) \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\ \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\ \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1+\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}}{(1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}} \right], -7+4\sqrt{3} \right] \right) / \\ \left( 5 b^{4/3} d \sqrt{a+bx} \sqrt{-\frac{(bc-ad)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right)$$

Result (type 5, 93 leaves):

$$\frac{1}{5bd\sqrt{a+bx}} \\ 6(c+dx)^{1/3} \left( d(a+bx) + (bc-ad) \sqrt{\frac{d(a+bx)}{-bc+ad}} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad} \right] \right)$$

**Problem 1562: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{1/3}}{(a+bx)^{3/2}} dx$$

Optimal (type 4, 366 leaves, 3 steps):

$$-\frac{2(c+dx)^{1/3}}{b\sqrt{a+bx}} - \left( 4 \sqrt{2-\sqrt{3}} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\ \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\ \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1+\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}}{(1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}} \right], -7+4\sqrt{3} \right] \right) / \\ \left( 3^{1/4} b^{4/3} \sqrt{a+bx} \sqrt{-\frac{(bc-ad)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right)$$

Result (type 5, 74 leaves):

$$\frac{2 (c + d x)^{1/3} \left( -1 + \sqrt{\frac{d (a + b x)}{-b c + a d}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)}{b \sqrt{a + b x}}$$

**Problem 1563: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + d x)^{1/3}}{(a + b x)^{5/2}} dx$$

Optimal (type 4, 417 leaves, 4 steps):

$$\begin{aligned} & -\frac{2 (c + d x)^{1/3}}{3 b (a + b x)^{3/2}} - \frac{4 d (c + d x)^{1/3}}{9 b (b c - a d) \sqrt{a + b x}} + \left( 4 \sqrt{2 - \sqrt{3}} d \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \right. \\ & \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \\ & \left. \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{(1 + \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}}{(1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3}} \right], -7 + 4 \sqrt{3} \right] \right) / \\ & \left( 9 \times 3^{1/4} b^{4/3} (b c - a d) \sqrt{a + b x} \sqrt{\frac{(b c - a d)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (1 - \sqrt{3}) (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)^2}} \right) \end{aligned}$$

Result (type 5, 104 leaves):

$$\begin{aligned} & \left( 2 (c + d x)^{1/3} \right. \\ & \left. \left( 3 b c - a d + 2 b d x + d (a + b x) \sqrt{\frac{d (a + b x)}{-b c + a d}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d} \right] \right) \right) / \\ & (9 b (-b c + a d) (a + b x)^{3/2}) \end{aligned}$$

**Problem 1564: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + d x)^{1/3}}{(a + b x)^{7/2}} dx$$

Optimal (type 4, 457 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{2(c+dx)^{1/3}}{5b(a+bx)^{5/2}} - \frac{4d(c+dx)^{1/3}}{45b(bc-ad)(a+bx)^{3/2}} + \\
 & \frac{28d^2(c+dx)^{1/3}}{135b(bc-ad)^2\sqrt{a+bx}} - \left( 28\sqrt{2-\sqrt{3}}d^2((bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}) \right. \\
 & \left. \sqrt{\frac{(bc-ad)^{2/3}+b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3}+b^{2/3}(c+dx)^{2/3}}{((1-\sqrt{3})(bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3})^2}} \right. \\
 & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1+\sqrt{3})(bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}}{(1-\sqrt{3})(bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}}\right], -7+4\sqrt{3}\right] \right) / \\
 & \left( 135 \times 3^{1/4} b^{4/3} (bc-ad)^2 \sqrt{a+bx} \sqrt{-\frac{(bc-ad)^{1/3}((bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3})}{((1-\sqrt{3})(bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3})^2}} \right)
 \end{aligned}$$

Result (type 5, 140 leaves):

$$\begin{aligned}
 & \left( 2(c+dx)^{1/3} \right. \\
 & \left. \left( -7a^2d^2 + 2abd(24c+17dx) + b^2(-27c^2 - 6cdx + 14d^2x^2) + 7d^2(a+bx)^2 \sqrt{\frac{d(a+bx)}{-bc+ad}} \right. \right. \\
 & \left. \left. \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) / (135b(bc-ad)^2(a+bx)^{5/2})
 \end{aligned}$$

**Problem 1565: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{1/3}} dx$$

Optimal (type 4, 839 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{54 (bc - ad) \sqrt{a+bx} (c+dx)^{2/3}}{91 d^2} + \frac{6 (a+bx)^{3/2} (c+dx)^{2/3}}{13 d} - \\
 & \frac{162 (bc - ad)^2 \sqrt{a+bx}}{91 b^{2/3} d^2 \left( (1 - \sqrt{3}) (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)} + \\
 & \left( 81 \times 3^{1/4} \sqrt{2 + \sqrt{3}} (bc - ad)^{7/3} \left( (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\
 & \sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (1 - \sqrt{3}) (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \\
 & \left. \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1 + \sqrt{3}) (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3}}{(1 - \sqrt{3}) (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3}} \right], -7 + 4 \sqrt{3} \right] \right) / \\
 & \left( 91 b^{2/3} d^3 \sqrt{a+bx} \sqrt{-\frac{(bc - ad)^{1/3} \left( (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (1 - \sqrt{3}) (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right) - \\
 & \left( 54 \sqrt{2} 3^{3/4} (bc - ad)^{7/3} \left( (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\
 & \sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (1 - \sqrt{3}) (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \\
 & \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 + \sqrt{3}) (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3}}{(1 - \sqrt{3}) (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3}} \right], -7 + 4 \sqrt{3} \right] \right) / \\
 & \left( 91 b^{2/3} d^3 \sqrt{a+bx} \sqrt{-\frac{(bc - ad)^{1/3} \left( (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (1 - \sqrt{3}) (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right)
 \end{aligned}$$

Result (type 5, 108 leaves):

$$\begin{aligned}
 & \frac{1}{182 d^3 \sqrt{a+bx}} 3 (c+dx)^{2/3} \left( 4 d (a+bx) (-9bc + 16ad + 7bdx) + \right. \\
 & \left. 27 (bc - ad)^2 \sqrt{\frac{d(a+bx)}{-bc+ad}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \frac{b(c+dx)}{bc-ad} \right] \right)
 \end{aligned}$$

### Problem 1566: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{1/3}} dx$$

Optimal (type 4, 804 leaves, 5 steps):

$$\frac{6\sqrt{a+bx}(c+dx)^{2/3}}{7d} + \frac{18(bc-ad)\sqrt{a+bx}}{7b^{2/3}d\left((1-\sqrt{3})(bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}\right)} - \left( 9 \times 3^{1/4} \sqrt{2+\sqrt{3}} (bc-ad)^{4/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{(1+\sqrt{3})(bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}}{(1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}}\right], -7+4\sqrt{3}\right] \right) / \left( 7b^{2/3}d^2\sqrt{a+bx} \sqrt{\frac{(bc-ad)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right) + \left( 6\sqrt{2} 3^{3/4} (bc-ad)^{4/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{(1+\sqrt{3})(bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}}{(1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}}\right], -7+4\sqrt{3}\right] \right) / \left( 7b^{2/3}d^2\sqrt{a+bx} \sqrt{\frac{(bc-ad)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right)$$

Result (type 5, 77 leaves):

$$\frac{3\sqrt{a+bx}(c+dx)^{2/3} \left( 4 + \frac{{}_3\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \frac{b(c+dx)}{bc-ad}\right]}{\sqrt{\frac{d(a+bx)}{-bc-ad}}} \right)}{14d}$$



### Problem 1567: Result unnecessarily involves higher level functions.

$$\int \frac{1}{\sqrt{a+bx} (c+dx)^{1/3}} dx$$

Optimal (type 4, 762 leaves, 4 steps):

$$\begin{aligned} & -\frac{6\sqrt{a+bx}}{b^{2/3} \left( (1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)} + \\ & \left( 3 \times 3^{1/4} \sqrt{2+\sqrt{3}} (bc-ad)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\ & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\ & \left. \text{EllipticE} \left[ \text{ArcSin} \left[ \frac{(1+\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}}{(1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}} \right], -7+4\sqrt{3} \right] \right) / \\ & \left( b^{2/3} d \sqrt{a+bx} \sqrt{\frac{(bc-ad)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right) - \\ & \left( 2\sqrt{2} 3^{3/4} (bc-ad)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\ & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\ & \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1+\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}}{(1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}} \right], -7+4\sqrt{3} \right] \right) / \\ & \left( b^{2/3} d \sqrt{a+bx} \sqrt{\frac{(bc-ad)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right) \end{aligned}$$

Result (type 5, 73 leaves):

$$\frac{3 \sqrt{\frac{d(a+bx)}{-bc+ad}} (c+dx)^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \frac{b(c+dx)}{bc-ad} \right]}{2d\sqrt{a+bx}}$$

### Problem 1568: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+bx)^{3/2} (c+dx)^{1/3}} dx$$

Optimal (type 4, 796 leaves, 5 steps):

$$\begin{aligned} & -\frac{2(c+dx)^{2/3}}{(bc-ad)\sqrt{a+bx}} - \frac{2d\sqrt{a+bx}}{b^{2/3}(bc-ad)\left((1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)} + \\ & \left( 3^{1/4}\sqrt{2+\sqrt{3}}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right) \right. \\ & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left((1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)^2}} \right. \\ & \left. \text{EllipticE}\left[\text{ArcSin}\left[\frac{(1+\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}}{(1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}}\right], -7+4\sqrt{3}\right] \right) / \\ & \left( b^{2/3}(bc-ad)^{2/3}\sqrt{a+bx} \sqrt{-\frac{(bc-ad)^{1/3}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)}{\left((1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)^2}} \right) - \\ & \left( 2\sqrt{2}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right) \right. \\ & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left((1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)^2}} \right. \\ & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1+\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}}{(1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}}\right], -7+4\sqrt{3}\right] \right) / \\ & \left( 3^{1/4}b^{2/3}(bc-ad)^{2/3}\sqrt{a+bx} \sqrt{-\frac{(bc-ad)^{1/3}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)}{\left((1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)^2}} \right) \end{aligned}$$

Result (type 5, 83 leaves):

$$\frac{(c+dx)^{2/3} \left( -4 + \sqrt{\frac{d(a+bx)}{-bc+ad}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \frac{b(c+dx)}{bc-ad}\right] \right)}{2(bc-ad)\sqrt{a+bx}}$$

**Problem 1569: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{5/2} (c+dx)^{1/3}} dx$$

Optimal (type 4, 842 leaves, 6 steps):

$$\begin{aligned} & -\frac{2(c+dx)^{2/3}}{3(bc-ad)(a+bx)^{3/2}} + \frac{10d(c+dx)^{2/3}}{9(bc-ad)^2\sqrt{a+bx}} + \\ & \frac{10d^2\sqrt{a+bx}}{9b^{2/3}(bc-ad)^2\left((1-\sqrt{3})(bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}\right)} - \\ & \left(5\sqrt{2+\sqrt{3}}d\left((bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}\right)\right. \\ & \left.\sqrt{\frac{(bc-ad)^{2/3}+b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3}+b^{2/3}(c+dx)^{2/3}}{\left((1-\sqrt{3})(bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}\right)^2}}\right. \\ & \left.\text{EllipticE}\left[\text{ArcSin}\left[\frac{(1+\sqrt{3})(bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}}{(1-\sqrt{3})(bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}}\right], -7+4\sqrt{3}\right]\right) / \\ & \left(3\times 3^{3/4}b^{2/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\frac{(bc-ad)^{1/3}\left((bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}\right)}{\left((1-\sqrt{3})(bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}\right)^2}}\right) + \\ & \left(10\sqrt{2}d\left((bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}\right)\right. \\ & \left.\sqrt{\frac{(bc-ad)^{2/3}+b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3}+b^{2/3}(c+dx)^{2/3}}{\left((1-\sqrt{3})(bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}\right)^2}}\right. \\ & \left.\text{EllipticF}\left[\text{ArcSin}\left[\frac{(1+\sqrt{3})(bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}}{(1-\sqrt{3})(bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}}\right], -7+4\sqrt{3}\right]\right) / \\ & \left(9\times 3^{1/4}b^{2/3}(bc-ad)^{5/3}\sqrt{a+bx}\sqrt{-\frac{(bc-ad)^{1/3}\left((bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}\right)}{\left((1-\sqrt{3})(bc-ad)^{1/3}-b^{1/3}(c+dx)^{1/3}\right)^2}}\right) \end{aligned}$$

Result (type 5, 105 leaves):

$$\left( (c+dx)^{2/3} \left( 4(-3bc+8ad+5bdx) - 5d(a+bx) \sqrt{\frac{d(a+bx)}{-bc+ad}} \right. \right. \\ \left. \left. \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2}{3}, \frac{5}{3}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) / (18(bc-ad)^2(a+bx)^{3/2})$$

**Problem 1570: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{2/3}} dx$$

Optimal (type 4, 416 leaves, 4 steps):

$$-\frac{54(bc-ad)\sqrt{a+bx}(c+dx)^{1/3}}{55d^2} + \frac{6(a+bx)^{3/2}(c+dx)^{1/3}}{11d} - \\ \left( 54 \times 3^{3/4} \sqrt{2-\sqrt{3}} (bc-ad)^2 \left( (bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right) \right. \\ \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left( (1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right)^2}} \right. \\ \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1+\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}}{(1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}}\right], -7+4\sqrt{3}\right] \right) / \\ \left( 55b^{1/3}d^3\sqrt{a+bx} \sqrt{-\frac{(bc-ad)^{1/3}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)}{\left( (1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right)^2}} \right)$$

Result (type 5, 108 leaves):

$$\frac{1}{55d^3\sqrt{a+bx}} 3(c+dx)^{1/3} \left( 2d(a+bx)(-9bc+14ad+5bdx) + \right. \\ \left. 27(bc-ad)^2 \sqrt{\frac{d(a+bx)}{-bc+ad}} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

**Problem 1571: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{2/3}} dx$$

Optimal (type 4, 381 leaves, 3 steps):

$$\frac{6 \sqrt{a+bx} (c+dx)^{1/3}}{5d} + \left( 6 \times 3^{3/4} \sqrt{2-\sqrt{3}} (bc-ad) \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\ \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\ \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1+\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}}{(1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}} \right], -7+4\sqrt{3} \right] \right) / \\ \left( 5 b^{1/3} d^2 \sqrt{a+bx} \sqrt{-\frac{(bc-ad)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right)$$

Result (type 5, 77 leaves):

$$\frac{3 \sqrt{a+bx} (c+dx)^{1/3} \left( 2 + \frac{{}_3\text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad} \right]}{\sqrt{\frac{d(a+bx)}{-bc+ad}}} \right)}{5d}$$

**Problem 1572: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{a+bx} (c+dx)^{2/3}} dx$$

Optimal (type 4, 345 leaves, 2 steps):

$$- \left( \left( 2 \times 3^{3/4} \sqrt{2-\sqrt{3}} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \right. \\ \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\ \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1+\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}}{(1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3}} \right], -7+4\sqrt{3} \right] \right) / \\ \left( b^{1/3} d \sqrt{a+bx} \sqrt{-\frac{(bc-ad)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (1-\sqrt{3}) (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)^2}} \right)$$

Result (type 5, 71 leaves):

$$\frac{3 \sqrt{\frac{d(a+bx)}{-bc+ad}} (c+dx)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad}\right]}{d \sqrt{a+bx}}$$

**Problem 1573: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{3/2} (c+dx)^{2/3}} dx$$

Optimal (type 4, 383 leaves, 3 steps):

$$\begin{aligned} & -\frac{2(c+dx)^{1/3}}{(bc-ad)\sqrt{a+bx}} + \left( 2\sqrt{2-\sqrt{3}} \left( (bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right) \right. \\ & \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left( (1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right)^2}} \\ & \left. \text{EllipticF}\left[\text{ArcSin}\left[\frac{(1+\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}}{(1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}}\right], -7+4\sqrt{3}\right] \right) / \\ & \left( 3^{1/4} b^{1/3} (bc-ad) \sqrt{a+bx} \sqrt{-\frac{(bc-ad)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right)}{\left( (1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right)^2}} \right) \end{aligned}$$

Result (type 5, 81 leaves):

$$\frac{(c+dx)^{1/3} \left( 2 + \sqrt{\frac{d(a+bx)}{-bc+ad}} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad}\right] \right)}{(bc-ad)\sqrt{a+bx}}$$

**Problem 1574: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{5/2} (c+dx)^{2/3}} dx$$

Optimal (type 4, 421 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{2(c+dx)^{1/3}}{3(bc-ad)(a+bx)^{3/2}} + \frac{14d(c+dx)^{1/3}}{9(bc-ad)^2\sqrt{a+bx}} - \left( 14\sqrt{2-\sqrt{3}}d \left( (bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right) \right. \\
 & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left( (1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1+\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}}{(1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}} \right], -7+4\sqrt{3} \right] \right) / \\
 & \left( 9 \times 3^{1/4} b^{1/3} (bc-ad)^2 \sqrt{a+bx} \sqrt{-\frac{(bc-ad)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right)}{\left( (1-\sqrt{3})(bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right)^2}} \right)
 \end{aligned}$$

Result (type 5, 102 leaves):

$$\left( (c+dx)^{1/3} \left( -6bc + 20ad + 14bdx + 7d(a+bx) \sqrt{\frac{d(a+bx)}{-bc+ad}} \right. \right. \\
 \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{2}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad} \right] \right) \right) / \left( 9(bc-ad)^2(a+bx)^{3/2} \right)$$

**Problem 1575: Result unnecessarily involves higher level functions.**

$$\int (a+bx)^{2/3} (c+dx)^{1/3} dx$$

Optimal (type 3, 219 leaves, 3 steps):

$$\begin{aligned}
 & \frac{(bc-ad)(a+bx)^{2/3}(c+dx)^{1/3}}{6bd} + \\
 & \frac{(a+bx)^{5/3}(c+dx)^{1/3}}{2b} + \frac{(bc-ad)^2 \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2d^{1/3}(a+bx)^{1/3}}{\sqrt{3}b^{1/3}(c+dx)^{1/3}} \right]}{3\sqrt{3}b^{4/3}d^{5/3}} + \\
 & \frac{(bc-ad)^2 \text{Log}[c+dx]}{18b^{4/3}d^{5/3}} + \frac{(bc-ad)^2 \text{Log} \left[ -1 + \frac{d^{1/3}(a+bx)^{1/3}}{b^{1/3}(c+dx)^{1/3}} \right]}{6b^{4/3}d^{5/3}}
 \end{aligned}$$

Result (type 5, 109 leaves):

$$\begin{aligned}
 & \frac{1}{6bd^2(a+bx)^{1/3}} (c+dx)^{1/3} \left( d(a+bx)(2ad+b(c+3dx)) - \right. \\
 & \left. 2(bc-ad)^2 \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad} \right] \right)
 \end{aligned}$$

### Problem 1576: Result unnecessarily involves higher level functions.

$$\int \frac{(c+dx)^{1/3}}{(a+bx)^{1/3}} dx$$

Optimal (type 3, 172 leaves, 2 steps):

$$\frac{(a+bx)^{2/3} (c+dx)^{1/3}}{b} - \frac{(bc-ad) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2d^{1/3}(a+bx)^{1/3}}{\sqrt{3}b^{1/3}(c+dx)^{1/3}}\right]}{\sqrt{3}b^{4/3}d^{2/3}} - \frac{(bc-ad) \operatorname{Log}[c+dx]}{6b^{4/3}d^{2/3}} - \frac{(bc-ad) \operatorname{Log}\left[-1 + \frac{d^{1/3}(a+bx)^{1/3}}{b^{1/3}(c+dx)^{1/3}}\right]}{2b^{4/3}d^{2/3}}$$

Result (type 5, 90 leaves):

$$\frac{1}{bd(a+bx)^{1/3}} (c+dx)^{1/3} \left( d(a+bx) + (bc-ad) \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

### Problem 1577: Result unnecessarily involves higher level functions.

$$\int \frac{(c+dx)^{1/3}}{(a+bx)^{4/3}} dx$$

Optimal (type 3, 149 leaves, 2 steps):

$$\frac{3(c+dx)^{1/3}}{b(a+bx)^{1/3}} - \frac{\sqrt{3}d^{1/3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2d^{1/3}(a+bx)^{1/3}}{\sqrt{3}b^{1/3}(c+dx)^{1/3}}\right]}{b^{4/3}} - \frac{d^{1/3} \operatorname{Log}[c+dx]}{2b^{4/3}} - \frac{3d^{1/3} \operatorname{Log}\left[-1 + \frac{d^{1/3}(a+bx)^{1/3}}{b^{1/3}(c+dx)^{1/3}}\right]}{2b^{4/3}}$$

Result (type 5, 74 leaves):

$$\frac{1}{b(a+bx)^{1/3}} (c+dx)^{1/3} \left( -1 + \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

### Problem 1582: Result unnecessarily involves higher level functions.

$$\int (a+bx)^{4/3} (c+dx)^{1/3} dx$$

Optimal (type 4, 655 leaves, 6 steps):



$$\begin{aligned}
 & - \frac{3 (b c - a d)^2 (a + b x)^{1/3} (c + d x)^{1/3}}{20 b d^2} + \frac{3 (b c - a d) (a + b x)^{4/3} (c + d x)^{1/3}}{40 b d} + \\
 & \frac{3 (a + b x)^{7/3} (c + d x)^{1/3}}{8 b} + \left( 3^{3/4} \sqrt{2 + \sqrt{3}} (b c - a d)^3 ((a + b x) (c + d x))^{2/3} \right. \\
 & \sqrt{(b c + a d + 2 b d x)^2 ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3})} \\
 & \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + \right. \\
 & \quad \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) /} \\
 & \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \text{EllipticF} [ \\
 & \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}, -7 - 4 \sqrt{3} \right] \Bigg) / \\
 & \left( 10 \times 2^{2/3} b^{4/3} d^{7/3} (a + b x)^{2/3} (c + d x)^{2/3} (b c + a d + 2 b d x) \right. \\
 & \left. \sqrt{\left( (b c - a d)^{2/3} ((b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) /} \right. \\
 & \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \sqrt{(a d + b (c + 2 d x))^2} \right)
 \end{aligned}$$

Result (type 5, 140 leaves):

$$\begin{aligned}
 & - \left( \left( 3 (c + d x)^{1/3} \left( -d (a + b x) (2 a^2 d^2 + a b d (5 c + 9 d x) + b^2 (-2 c^2 + c d x + 5 d^2 x^2)) - 2 (b c - a d)^3 \right. \right. \right. \\
 & \quad \left. \left. \left. \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d} \right] \right) \right) \Bigg) / \left( 40 b d^3 (a + b x)^{2/3} \right)
 \end{aligned}$$

**Problem 1583: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^{1/3} (c + d x)^{1/3} dx$$

Optimal (type 4, 617 leaves, 5 steps):

$$\frac{3 (bc - ad) (a + bx)^{1/3} (c + dx)^{1/3}}{10bd} + \frac{3 (a + bx)^{4/3} (c + dx)^{1/3}}{5b} -$$

$$\left( 3^{3/4} \sqrt{2 + \sqrt{3}} (bc - ad)^2 ((a + bx) (c + dx))^{2/3} \right.$$

$$\sqrt{(bc + ad + 2bdx)^2 ((bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3})}$$

$$\sqrt{\left( (bc - ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc - ad)^{2/3} ((a + bx) (c + dx))^{1/3} + \right.$$

$$\left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + bx) (c + dx))^{2/3} \right) /}$$

$$\left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right)^2 \text{EllipticF} \left[ \right.$$

$$\text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3}}{(1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3}}, -7 - 4\sqrt{3} \right] \left. / \right)$$

$$\left( 5 \times 2^{2/3} b^{4/3} d^{4/3} (a + bx)^{2/3} (c + dx)^{2/3} (bc + ad + 2bdx) \right.$$

$$\sqrt{\left( (bc - ad)^{2/3} ((bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) /}$$

$$\left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right)^2 \sqrt{(ad + b(c + 2dx))^2} \right)$$

Result (type 5, 108 leaves):

$$\left( 3 (c + dx)^{1/3} \left( d (a + bx) (ad + b(c + 2dx)) - (bc - ad)^2 \left( \frac{d(a + bx)}{-bc + ad} \right)^{2/3} \right. \right.$$

$$\left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b(c + dx)}{bc - ad} \right] \right) \right) / (10bd^2 (a + bx)^{2/3})$$

**Problem 1584: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + dx)^{1/3}}{(a + bx)^{2/3}} dx$$

Optimal (type 4, 576 leaves, 4 steps):

$$\begin{aligned}
 & \frac{3 (a+bx)^{1/3} (c+dx)^{1/3}}{2b} + \left( 3^{3/4} \sqrt{2+\sqrt{3}} (bc-ad) ((a+bx)(c+dx))^{2/3} \right. \\
 & \quad \sqrt{(bc+ad+2bdx)^2} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \\
 & \quad \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + \right. \\
 & \quad \quad \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) /} \\
 & \quad \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \text{EllipticF} \left[ \right. \\
 & \quad \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3}) (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3}) (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}} \right], -7-4\sqrt{3} \right] \left. \right) / \\
 & \quad \left( 2^{2/3} b^{4/3} d^{1/3} (a+bx)^{2/3} (c+dx)^{2/3} (bc+ad+2bdx) \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) /} \right. \\
 & \quad \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 93 leaves):

$$\begin{aligned}
 & \frac{1}{2bd(a+bx)^{2/3}} \\
 & 3(c+dx)^{1/3} \left( d(a+bx) + (bc-ad) \left( \frac{d(a+bx)}{-bc+ad} \right)^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad} \right] \right)
 \end{aligned}$$

**Problem 1585: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{1/3}}{(a+bx)^{5/3}} dx$$

Optimal (type 4, 568 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{3(c+dx)^{1/3}}{2b(a+bx)^{2/3}} + \left( 3^{3/4} \sqrt{2+\sqrt{3}} d^{2/3} ((a+bx)(c+dx))^{2/3} \right. \\
 & \quad \sqrt{(bc+ad+2bdx)^2} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \\
 & \quad \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + \right. \\
 & \quad \quad \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) /} \\
 & \quad \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \text{EllipticF} \left[ \right. \\
 & \quad \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right] / \\
 & \quad \left( 2^{2/3} b^{4/3} (a+bx)^{2/3} (c+dx)^{2/3} (bc+ad+2bdx) \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) /} \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 76 leaves):

$$\frac{1}{2b(a+bx)^{2/3}} (c+dx)^{1/3} \left( -1 + \left( \frac{d(a+bx)}{-bc+ad} \right)^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad} \right] \right)$$

**Problem 1586: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{1/3}}{(a+bx)^{8/3}} dx$$

Optimal (type 4, 617 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{3(c+dx)^{1/3}}{5b(a+bx)^{5/3}} - \frac{3d(c+dx)^{1/3}}{10b(bc-ad)(a+bx)^{2/3}} - \\
 & \left( 3^{3/4} \sqrt{2+\sqrt{3}} d^{5/3} ((a+bx)(c+dx))^{2/3} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \quad \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \\
 & \quad \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + \right. \\
 & \quad \quad \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) /} \\
 & \quad \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \text{EllipticF}\left[ \right. \\
 & \quad \left. \text{ArcSin}\left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}} \right], -7-4\sqrt{3} \right] \left. \right) / \\
 & \quad \left( 5 \times 2^{2/3} b^{4/3} (bc-ad)(a+bx)^{2/3} (c+dx)^{2/3} (bc+ad+2bdx) \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{2/3} ((bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) /} \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 103 leaves):

$$\begin{aligned}
 & \left( 3(c+dx)^{1/3} \right. \\
 & \quad \left. \left( 2bc-ad+bdx+d(a+bx) \left( \frac{d(a+bx)}{-bc+ad} \right)^{2/3} \text{Hypergeometric2F1}\left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad} \right] \right) \right) / \\
 & \quad \left( 10b(-bc+ad)(a+bx)^{5/3} \right)
 \end{aligned}$$

**Problem 1587: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{4/3}}{(c+dx)^{1/3}} dx$$

Optimal (type 3, 216 leaves, 3 steps):

$$\begin{aligned}
 & -\frac{2(bc-ad)(a+bx)^{1/3}(c+dx)^{2/3}}{3d^2} + \\
 & \quad \frac{(a+bx)^{4/3}(c+dx)^{2/3}}{2d} - \frac{2(bc-ad)^2 \text{ArcTan}\left[ \frac{1}{\sqrt{3}} + \frac{2b^{1/3}(c+dx)^{1/3}}{\sqrt{3}d^{1/3}(a+bx)^{1/3}} \right]}{3\sqrt{3}b^{2/3}d^{7/3}} - \\
 & \quad \frac{(bc-ad)^2 \text{Log}[a+bx]}{9b^{2/3}d^{7/3}} - \frac{(bc-ad)^2 \text{Log}\left[-1 + \frac{b^{1/3}(c+dx)^{1/3}}{d^{1/3}(a+bx)^{1/3}}\right]}{3b^{2/3}d^{7/3}}
 \end{aligned}$$

Result (type 5, 107 leaves):

$$\frac{1}{6 d^3 (a+b x)^{2/3}} (c+d x)^{2/3} \left( d (a+b x) (-4 b c+7 a d+3 b d x)+2 (b c-a d)^2 \left( \frac{d (a+b x)}{-b c+a d} \right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c-a d}\right]\right)$$

**Problem 1588: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+b x)^{1/3}}{(c+d x)^{1/3}} dx$$

Optimal (type 3, 171 leaves, 2 steps):

$$\frac{(a+b x)^{1/3} (c+d x)^{2/3}}{d} + \frac{(b c-a d) \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{\sqrt{3} b^{2/3} d^{4/3}} + \frac{(b c-a d) \text{Log}[a+b x]}{6 b^{2/3} d^{4/3}} + \frac{(b c-a d) \text{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{2 b^{2/3} d^{4/3}}$$

Result (type 5, 76 leaves):

$$\frac{(a+b x)^{1/3} (c+d x)^{2/3} \left( 2 + \frac{\text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c-a d}\right]}{\left(\frac{d (a+b x)}{-b c+a d}\right)^{1/3}} \right)}{2 d}$$

**Problem 1589: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+b x)^{2/3} (c+d x)^{1/3}} dx$$

Optimal (type 3, 126 leaves, 1 step):

$$-\frac{\sqrt{3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{b^{2/3} d^{1/3}} - \frac{\text{Log}[a+b x]}{2 b^{2/3} d^{1/3}} - \frac{3 \text{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{2 b^{2/3} d^{1/3}}$$

Result (type 5, 73 leaves):

$$\frac{3 \left( \frac{d (a+b x)}{-b c+a d} \right)^{2/3} (c+d x)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c-a d}\right]}{2 d (a+b x)^{2/3}}$$

**Problem 1594: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+b x)^{8/3}}{(c+d x)^{1/3}} dx$$

Optimal (type 4, 1365 leaves, 8 steps):

$$\begin{aligned}
 & \frac{3 (bc - ad)^2 (a + bx)^{2/3} (c + dx)^{2/3}}{7 d^3} - \\
 & \frac{12 (bc - ad) (a + bx)^{5/3} (c + dx)^{2/3}}{35 d^2} + \frac{3 (a + bx)^{8/3} (c + dx)^{2/3}}{10 d} - \\
 & \left( 3 \times 2^{2/3} (bc - ad)^3 ((a + bx) (c + dx))^{1/3} \sqrt{(bc + ad + 2bdx)^2} \sqrt{(ad + b(c + 2dx))^2} \right) / \\
 & \left( 7 b^{2/3} d^{11/3} (a + bx)^{1/3} (c + dx)^{1/3} (bc + ad + 2bdx) \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) \right) + \\
 & \left( 3 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (bc - ad)^{11/3} ((a + bx) (c + dx))^{1/3} \sqrt{(bc + ad + 2bdx)^2} \right. \\
 & \quad \left( (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) \sqrt{\left( (bc - ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} \right.} \\
 & \quad \left. (bc - ad)^{2/3} ((a + bx) (c + dx))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + bx) (c + dx))^{2/3} \right) / \\
 & \quad \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right)^2 \text{EllipticE} \left[ \right. \\
 & \quad \left. \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3}}{(1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3}}, -7 - 4\sqrt{3} \right] \right) / \\
 & \left( 7 \times 2^{1/3} b^{2/3} d^{11/3} (a + bx)^{1/3} (c + dx)^{1/3} (bc + ad + 2bdx) \right. \\
 & \quad \sqrt{\left( (bc - ad)^{2/3} \left( (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) \right) /} \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right)^2 \sqrt{(ad + b(c + 2dx))^2} \right) - \\
 & \left( 2 \times 2^{1/6} \times 3^{3/4} (bc - ad)^{11/3} ((a + bx) (c + dx))^{1/3} \sqrt{(bc + ad + 2bdx)^2} \right. \\
 & \quad \left( (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) \\
 & \quad \sqrt{\left( (bc - ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc - ad)^{2/3} ((a + bx) (c + dx))^{1/3} + \right.} \\
 & \quad \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + bx) (c + dx))^{2/3} \right) /} \\
 & \quad \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right)^2 \text{EllipticF} \left[ \right. \\
 & \quad \left. \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3}}{(1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3}}, -7 - 4\sqrt{3} \right] \right) / \\
 & \left( 7 b^{2/3} d^{11/3} (a + bx)^{1/3} (c + dx)^{1/3} (bc + ad + 2bdx) \right. \\
 & \quad \sqrt{\left( (bc - ad)^{2/3} \left( (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) \right) /} \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right)^2 \sqrt{(ad + b(c + 2dx))^2} \right)
 \end{aligned}$$

Result(type 5, 138 leaves):

$$\frac{1}{70 d^4 (a + b x)^{1/3}} - 3 (c + d x)^{2/3} \left( d (a + b x) (25 a^2 d^2 + 2 a b d (-14 c + 11 d x) + b^2 (10 c^2 - 8 c d x + 7 d^2 x^2)) - 10 (b c - a d)^3 \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

**Problem 1595: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{5/3}}{(c + d x)^{1/3}} dx$$

Optimal (type 4, 1330 leaves, 7 steps):



$$\begin{aligned}
 & - \frac{15 (bc - ad) (a + bx)^{2/3} (c + dx)^{2/3}}{28 d^2} + \frac{3 (a + bx)^{5/3} (c + dx)^{2/3}}{7 d} + \\
 & \left( \frac{15 (bc - ad)^2 ((a + bx) (c + dx))^{1/3} \sqrt{(bc + ad + 2bdx)^2} \sqrt{(ad + b(c + 2dx))^2}}{14 \times 2^{1/3} b^{2/3} d^{8/3} (a + bx)^{1/3} (c + dx)^{1/3} (bc + ad + 2bdx)} \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) - \right. \\
 & \left( 15 \times 3^{1/4} \sqrt{2 - \sqrt{3}} (bc - ad)^{8/3} ((a + bx) (c + dx))^{1/3} \sqrt{(bc + ad + 2bdx)^2} \right. \\
 & \quad \left. \left( (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) \sqrt{\left( (bc - ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} \right. \right.} \\
 & \quad \left. \left. (bc - ad)^{2/3} ((a + bx) (c + dx))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + bx) (c + dx))^{2/3} \right) / \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right)^2 \right] \text{EllipticE} \left[ \right. \\
 & \quad \left. \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3}}{(1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3}} \right], -7 - 4\sqrt{3} \right] \left. \right] / \\
 & \left( 28 \times 2^{1/3} b^{2/3} d^{8/3} (a + bx)^{1/3} (c + dx)^{1/3} (bc + ad + 2bdx) \right. \\
 & \quad \left. \sqrt{\left( (bc - ad)^{2/3} ((bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) / \right.} \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right)^2 \right) \sqrt{(ad + b(c + 2dx))^2} + \\
 & \left( 5 \times 3^{3/4} (bc - ad)^{8/3} ((a + bx) (c + dx))^{1/3} \sqrt{(bc + ad + 2bdx)^2} \right. \\
 & \quad \left. \left( (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) \right. \\
 & \quad \left. \sqrt{\left( (bc - ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc - ad)^{2/3} ((a + bx) (c + dx))^{1/3} + \right. \right.} \\
 & \quad \left. \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + bx) (c + dx))^{2/3} \right) / \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right)^2 \right] \text{EllipticF} \left[ \right. \\
 & \quad \left. \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3}}{(1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3}} \right], -7 - 4\sqrt{3} \right] \left. \right] / \\
 & \left( 7 \times 2^{5/6} b^{2/3} d^{8/3} (a + bx)^{1/3} (c + dx)^{1/3} (bc + ad + 2bdx) \right. \\
 & \quad \left. \sqrt{\left( (bc - ad)^{2/3} ((bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) / \right.} \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right)^2 \right) \sqrt{(ad + b(c + 2dx))^2}
 \end{aligned}$$

Result (type 5, 107 leaves):

$$\begin{aligned}
 & \frac{1}{28 d^3 (a + bx)^{1/3}} 3 (c + dx)^{2/3} \left( d (a + bx) (-5bc + 9ad + 4bdx) + \right. \\
 & \quad \left. 5 (bc - ad)^2 \left( \frac{d (a + bx)}{-bc + ad} \right)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b(c + dx)}{bc - ad} \right] \right)
 \end{aligned}$$

### Problem 1596: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx)^{2/3}}{(c+dx)^{1/3}} dx$$

Optimal (type 4, 1293 leaves, 6 steps):

$$\frac{3 (a+bx)^{2/3} (c+dx)^{2/3}}{4d} - \left( \frac{3 (bc-ad) ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2}}{2 \times 2^{1/3} b^{2/3} d^{5/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx)} \right. \\ \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) + \right. \\ \left. \left( 3 \times 3^{1/4} \sqrt{2-\sqrt{3}} (bc-ad)^{5/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \right. \\ \left. \left. \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} \right. \right. \right. \\ \left. \left. \left. (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \right. \right. \\ \left. \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \text{EllipticE} \left[ \right. \\ \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3}) (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3}) (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right] \right) / \\ \left( 4 \times 2^{1/3} b^{2/3} d^{5/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\ \left. \sqrt{\left( (bc-ad)^{2/3} ((bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}) \right) / \right. \\ \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \sqrt{(ad+b(c+2dx))^2} - \\ \left( 3^{3/4} (bc-ad)^{5/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \\ \left. \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right. \\ \left. \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + \right. \right. \\ \left. \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \right. \\ \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \text{EllipticF} \left[ \right. \\ \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3}) (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3}) (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right] \right) / \\ \left( 2^{5/6} b^{2/3} d^{5/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\ \left. \sqrt{\left( (bc-ad)^{2/3} ((bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}) \right) / \right. \\ \left. \left( (1+\sqrt{3}) (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \sqrt{(ad+b(c+2dx))^2} \right)$$

Result (type 5, 76 leaves):

$$\frac{3 (a + b x)^{2/3} (c + d x)^{2/3} \left( 1 + \frac{\text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b(c+dx)}{b c - a d}\right]}{\left(\frac{d(a+bx)}{-b c + a d}\right)^{2/3}} \right)}{4 d}$$

Problem 1597: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{1/3} (c + d x)^{1/3}} dx$$

Optimal (type 4, 1257 leaves, 5 steps):

$$\begin{aligned}
 & \left( 3 \left( (a+bx)(c+dx) \right)^{1/3} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2} \right) / \\
 & \left( 2^{1/3} b^{2/3} d^{2/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\
 & \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) - \\
 & \left( 3 \times 3^{1/4} \sqrt{2-\sqrt{3}} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} \right.} \\
 & \left. (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \\
 & \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \text{EllipticE} \left[ \right. \\
 & \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right) / \\
 & \left( 2 \times 2^{1/3} b^{2/3} d^{2/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\
 & \left. \sqrt{\left( (bc-ad)^{2/3} ((bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}) \right) /} \right. \\
 & \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \sqrt{(ad+b(c+2dx))^2} \right) + \\
 & \left( 2^{1/6} \times 3^{3/4} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \\
 & \left. \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + \right.} \right. \\
 & \left. \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) /} \right. \\
 & \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \text{EllipticF} \left[ \right. \\
 & \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right) / \\
 & \left( b^{2/3} d^{2/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\
 & \left. \sqrt{\left( (bc-ad)^{2/3} ((bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}) \right) /} \right. \\
 & \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 73 leaves):

$$\frac{3 \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/3} (c+dx)^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b(c+dx)}{bc-ad} \right]}{2d(a+bx)^{1/3}}$$

### Problem 1598: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+bx)^{4/3} (c+dx)^{1/3}} dx$$

Optimal (type 4, 1297 leaves, 6 steps):

$$\begin{aligned} & -\frac{3(c+dx)^{2/3}}{(bc-ad)(a+bx)^{1/3}} + \\ & \left( 3d^{1/3}((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2} \right) / \\ & \left( 2^{1/3} b^{2/3} (bc-ad)(a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\ & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) - \right. \\ & \left. \left( 3 \times 3^{1/4} \sqrt{2-\sqrt{3}} d^{1/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \right. \\ & \quad \left. \left. \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} \right. \right. \right. \\ & \quad \left. \left. \left. (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \right. \right. \\ & \quad \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \text{EllipticE} \left[ \right. \\ & \quad \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right] / \\ & \left( 2 \times 2^{1/3} b^{2/3} (bc-ad)^{1/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\ & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) / \right. \\ & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \sqrt{(ad+b(c+2dx))^2} + \\ & \left( 2^{1/6} \times 3^{3/4} d^{1/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \\ & \quad \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \\ & \quad \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + \right. \\ & \quad \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \\ & \quad \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \text{EllipticF} \left[ \right. \\ & \quad \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right] / \\ & \left( b^{2/3} (bc-ad)^{1/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\ & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) / \right. \\ & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \sqrt{(ad+b(c+2dx))^2} \end{aligned}$$

Result (type 5, 83 leaves):

$$\frac{\left(3 (c + d x)^{2/3} \left(-2 + \left(\frac{d (a + b x)}{-b c + a d}\right)^{1/3} \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d}\right]\right)\right)}{\left(2 (b c - a d) (a + b x)^{1/3}\right)}$$

**Problem 1599: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b x)^{7/3} (c + d x)^{1/3}} dx$$

Optimal (type 4, 1335 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{3(c+dx)^{2/3}}{4(bc-ad)(a+bx)^{4/3}} + \frac{3d(c+dx)^{2/3}}{2(bc-ad)^2(a+bx)^{1/3}} - \\
 & \left( \frac{3d^{4/3}((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2}}{2 \times 2^{1/3} b^{2/3} (bc-ad)^2 (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx)} \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) + \right. \\
 & \quad \left( 3 \times 3^{1/4} \sqrt{2-\sqrt{3}} d^{4/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \quad \left. \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} \right. \right. \\
 & \quad \left. \left. (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right)} \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \text{EllipticE} \left[ \right. \\
 & \quad \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right] \Bigg/ \\
 & \left( 4 \times 2^{1/3} b^{2/3} (bc-ad)^{4/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\
 & \quad \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right.} \\
 & \quad \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \sqrt{(ad+b(c+2dx))^2} \right) - \\
 & \left( 3^{3/4} d^{4/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \quad \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \\
 & \quad \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + \right.} \\
 & \quad \left. \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right)} \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \text{EllipticF} \left[ \right. \\
 & \quad \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right] \Bigg/ \\
 & \left( 2^{5/6} b^{2/3} (bc-ad)^{4/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\
 & \quad \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right.} \\
 & \quad \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 100 leaves):

$$\begin{aligned}
 & - \left( \left( 3(c+dx)^{2/3} \left( -3ad + b(c-2dx) + d(a+bx) \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/3} \right. \right. \right. \\
 & \quad \left. \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b(c+dx)}{bc-ad} \right] \right) \right) \right) \Bigg/ \left( 4(bc-ad)^2(a+bx)^{4/3} \right)
 \end{aligned}$$

### Problem 1600: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+bx)^{10/3} (c+dx)^{1/3}} dx$$

Optimal (type 4, 1372 leaves, 8 steps):

$$\begin{aligned} & -\frac{3(c+dx)^{2/3}}{7(bc-ad)(a+bx)^{7/3}} + \frac{15d(c+dx)^{2/3}}{28(bc-ad)^2(a+bx)^{4/3}} - \frac{15d^2(c+dx)^{2/3}}{14(bc-ad)^3(a+bx)^{1/3}} + \\ & \left( \frac{15d^{7/3}((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2}}{14 \times 2^{1/3} b^{2/3} (bc-ad)^3 (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx)} \right. \\ & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) - \right. \\ & \quad \left. \left( 15 \times 3^{1/4} \sqrt{2-\sqrt{3}} d^{7/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \right. \\ & \quad \left. \left. \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} \right. \right. \right. \\ & \quad \left. \left. \left. (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \right. \right. \\ & \quad \left. \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \text{EllipticE} \left[ \right. \right. \\ & \quad \left. \left. \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}} \right], -7-4\sqrt{3} \right] \right] \right) / \\ & \quad \left( 28 \times 2^{1/3} b^{2/3} (bc-ad)^{7/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\ & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) / \right. \\ & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \sqrt{(ad+b(c+2dx))^2} \right) + \\ & \quad \left( 5 \times 3^{3/4} d^{7/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \\ & \quad \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \\ & \quad \left. \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + \right. \right. \\ & \quad \left. \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \right. \\ & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \text{EllipticF} \left[ \right. \\ & \quad \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}} \right], -7-4\sqrt{3} \right] \right) / \\ & \quad \left( 7 \times 2^{5/6} b^{2/3} (bc-ad)^{7/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\ & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) / \right. \\ & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \sqrt{(ad+b(c+2dx))^2} \right) \end{aligned}$$



Result (type 5, 136 leaves):

$$\left( 3 (c + dx)^{2/3} \right. \\ \left. \left( -19 a^2 d^2 + a b d (13 c - 25 dx) + b^2 (-4 c^2 + 5 c dx - 10 d^2 x^2) + 5 d^2 (a + bx)^2 \left( \frac{d (a + bx)}{-bc + ad} \right)^{1/3} \right. \right. \\ \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + dx)}{bc - ad} \right] \right) \right) / \left( 28 (bc - ad)^3 (a + bx)^{7/3} \right)$$

**Problem 1601: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + bx)^{5/3}}{(c + dx)^{2/3}} dx$$

Optimal (type 3, 216 leaves, 3 steps):

$$- \frac{5 (bc - ad) (a + bx)^{2/3} (c + dx)^{1/3}}{6 d^2} + \\ \frac{(a + bx)^{5/3} (c + dx)^{1/3}}{2 d} - \frac{5 (bc - ad)^2 \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2 d^{1/3} (a + bx)^{1/3}}{\sqrt{3} b^{1/3} (c + dx)^{1/3}} \right]}{3 \sqrt{3} b^{1/3} d^{8/3}} - \\ \frac{5 (bc - ad)^2 \text{Log}[c + dx]}{18 b^{1/3} d^{8/3}} - \frac{5 (bc - ad)^2 \text{Log} \left[ -1 + \frac{d^{1/3} (a + bx)^{1/3}}{b^{1/3} (c + dx)^{1/3}} \right]}{6 b^{1/3} d^{8/3}}$$

Result (type 5, 107 leaves):

$$\frac{1}{6 d^3 (a + bx)^{1/3}} (c + dx)^{1/3} \left( d (a + bx) (-5 bc + 8 ad + 3 b dx) + \right. \\ \left. 10 (bc - ad)^2 \left( \frac{d (a + bx)}{-bc + ad} \right)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b (c + dx)}{bc - ad} \right] \right)$$

**Problem 1602: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + bx)^{2/3}}{(c + dx)^{2/3}} dx$$

Optimal (type 3, 169 leaves, 2 steps):

$$\frac{(a + bx)^{2/3} (c + dx)^{1/3}}{d} + \frac{2 (bc - ad) \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2 d^{1/3} (a + bx)^{1/3}}{\sqrt{3} b^{1/3} (c + dx)^{1/3}} \right]}{\sqrt{3} b^{1/3} d^{5/3}} + \\ \frac{(bc - ad) \text{Log}[c + dx]}{3 b^{1/3} d^{5/3}} + \frac{(bc - ad) \text{Log} \left[ -1 + \frac{d^{1/3} (a + bx)^{1/3}}{b^{1/3} (c + dx)^{1/3}} \right]}{b^{1/3} d^{5/3}}$$

Result (type 5, 74 leaves):

$$\frac{(a+bx)^{2/3} (c+dx)^{1/3} \left( 1 + \frac{2 \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad}\right]}{\left(\frac{d(a+bx)}{-bc+ad}\right)^{2/3}} \right)}{d}$$

**Problem 1603: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{1/3} (c+dx)^{2/3}} dx$$

Optimal (type 3, 126 leaves, 1 step):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2d^{1/3}(a+bx)^{1/3}}{\sqrt{3}b^{1/3}(c+dx)^{1/3}}\right]}{b^{1/3}d^{2/3}} - \frac{\operatorname{Log}[c+dx]}{2b^{1/3}d^{2/3}} - \frac{3 \operatorname{Log}\left[-1 + \frac{d^{1/3}(a+bx)^{1/3}}{b^{1/3}(c+dx)^{1/3}}\right]}{2b^{1/3}d^{2/3}}$$

Result (type 5, 71 leaves):

$$\frac{3 \left(\frac{d(a+bx)}{-bc+ad}\right)^{1/3} (c+dx)^{1/3} \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad}\right]}{d(a+bx)^{1/3}}$$

**Problem 1608: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{7/3}}{(c+dx)^{2/3}} dx$$

Optimal (type 4, 649 leaves, 6 steps):

$$\begin{aligned} & \frac{21(bc-ad)^2(a+bx)^{1/3}(c+dx)^{1/3}}{20d^3} - \\ & \frac{21(bc-ad)(a+bx)^{4/3}(c+dx)^{1/3}}{40d^2} + \frac{3(a+bx)^{7/3}(c+dx)^{1/3}}{8d} - \\ & \left( 7 \times 3^{3/4} \sqrt{2+\sqrt{3}} (bc-ad)^3 ((a+bx)(c+dx))^{2/3} \sqrt{(bc+ad+2bdx)^2} \right. \\ & \left. \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} \right.} \right. \\ & \left. \left. (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \right. \\ & \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \operatorname{EllipticF}\left[ \right. \\ & \left. \operatorname{ArcSin}\left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}} \right], -7-4\sqrt{3} \right] \right) / \\ & \left( 10 \times 2^{2/3} b^{1/3} d^{10/3} (a+bx)^{2/3} (c+dx)^{2/3} (bc+ad+2bdx) \right. \\ & \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) /} \right. \\ & \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \sqrt{(ad+b(c+2dx))^2} \right) \end{aligned}$$

Result (type 5, 137 leaves):

$$\frac{1}{40 d^4 (a + b x)^{2/3}} - 3 (c + d x)^{1/3} \left( d (a + b x) (26 a^2 d^2 + a b d (-35 c + 17 d x) + b^2 (14 c^2 - 7 c d x + 5 d^2 x^2)) - 14 (b c - a d)^3 \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

**Problem 1609: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{4/3}}{(c + d x)^{2/3}} dx$$

Optimal (type 4, 614 leaves, 5 steps):

$$\begin{aligned} & -\frac{6 (b c - a d) (a + b x)^{1/3} (c + d x)^{1/3}}{5 d^2} + \frac{3 (a + b x)^{4/3} (c + d x)^{1/3}}{5 d} + \\ & \left( 2 \times 2^{1/3} \times 3^{3/4} \sqrt{2 + \sqrt{3}} (b c - a d)^2 ((a + b x) (c + d x))^{2/3} \right. \\ & \quad \sqrt{(b c + a d + 2 b d x)^2} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \\ & \quad \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} ((a + b x) (c + d x))^{1/3} + \right. \\ & \quad \quad \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + b x) (c + d x))^{2/3} \right) /} \\ & \quad \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \text{EllipticF} \left[ \right. \\ & \quad \left. \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3}}, -7 - 4 \sqrt{3} \right] \right] / \\ & \left( 5 b^{1/3} d^{7/3} (a + b x)^{2/3} (c + d x)^{2/3} (b c + a d + 2 b d x) \right. \\ & \quad \left. \sqrt{\left( (b c - a d)^{2/3} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right) \right) /} \right. \\ & \quad \left. \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + b x) (c + d x))^{1/3} \right)^2 \sqrt{(a d + b (c + 2 d x))^2} \right) \end{aligned}$$

Result (type 5, 106 leaves):

$$\frac{1}{5 d^3 (a + b x)^{2/3}} - 3 (c + d x)^{1/3} \left( d (a + b x) (-2 b c + 3 a d + b d x) + 2 (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

**Problem 1610: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{1/3}}{(c + d x)^{2/3}} dx$$

Optimal (type 4, 577 leaves, 4 steps):

$$\frac{3 (a + b x)^{1/3} (c + d x)^{1/3}}{2 d} - \left( 3^{3/4} \sqrt{2 + \sqrt{3}} (b c - a d) \left( (a + b x) (c + d x) \right)^{2/3} \right. \\ \sqrt{(b c + a d + 2 b d x)^2 \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + b x) (c + d x) \right)^{1/3} \right)} \\ \sqrt{\left( (b c - a d)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (b c - a d)^{2/3} \left( (a + b x) (c + d x) \right)^{1/3} + \right. \\ \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} \left( (a + b x) (c + d x) \right)^{2/3} \right) /} \\ \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + b x) (c + d x) \right)^{1/3} \right)^2 \text{EllipticF} [ \\ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + b x) (c + d x) \right)^{1/3}}{(1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + b x) (c + d x) \right)^{1/3}}, -7 - 4 \sqrt{3} \right] \Bigg) / \\ \left( 2^{2/3} b^{1/3} d^{4/3} (a + b x)^{2/3} (c + d x)^{2/3} (b c + a d + 2 b d x) \right) \\ \sqrt{\left( (b c - a d)^{2/3} \left( (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + b x) (c + d x) \right)^{1/3} \right) \right) /} \\ \left( (1 + \sqrt{3}) (b c - a d)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + b x) (c + d x) \right)^{1/3} \right)^2 \sqrt{(a d + b (c + 2 d x))^2}$$

Result (type 5, 76 leaves):

$$\frac{3 (a + b x)^{1/3} (c + d x)^{1/3} \left( 1 + \frac{\text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b(c+dx)}{b c - a d} \right]}{\left( \frac{d(a-bx)}{-b c + a d} \right)^{1/3}} \right)}{2 d}$$

**Problem 1611: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b x)^{2/3} (c + d x)^{2/3}} dx$$

Optimal (type 4, 542 leaves, 3 steps):

$$\begin{aligned}
 & \left( 2^{1/3} \times 3^{3/4} \sqrt{2 + \sqrt{3}} \left( (a + bx)(c + dx) \right)^{2/3} \right. \\
 & \quad \sqrt{(bc + ad + 2bdx)^2 \left( (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + bx)(c + dx) \right)^{1/3} \right)} \\
 & \quad \sqrt{\left( (bc - ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc - ad)^{2/3} \left( (a + bx)(c + dx) \right)^{1/3} + \right. \\
 & \quad \quad \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} \left( (a + bx)(c + dx) \right)^{2/3} \right) /} \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + bx)(c + dx) \right)^{1/3} \right)^2 \right) \\
 & \quad \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + bx)(c + dx) \right)^{1/3}}{(1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + bx)(c + dx) \right)^{1/3}}, -7 - 4\sqrt{3} \right] \right] / \\
 & \quad \left( b^{1/3} d^{1/3} (a + bx)^{2/3} (c + dx)^{2/3} (bc + ad + 2bdx) \right. \\
 & \quad \sqrt{\left( (bc - ad)^{2/3} \left( (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + bx)(c + dx) \right)^{1/3} \right) \right) /} \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + bx)(c + dx) \right)^{1/3} \right)^2 \sqrt{(ad + b(c + 2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 71 leaves):

$$\frac{3 \left( \frac{d(a+bx)}{-bc+ad} \right)^{2/3} (c+dx)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad} \right]}{d (a+bx)^{2/3}}$$

**Problem 1612: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{5/3} (c+dx)^{2/3}} dx$$

Optimal (type 4, 586 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{3(c+dx)^{1/3}}{2(bc-ad)(a+bx)^{2/3}} - \left( 3^{3/4} \sqrt{2 + \sqrt{3}} d^{2/3} \left( (a + bx)(c + dx) \right)^{2/3} \right. \\
 & \quad \sqrt{(bc + ad + 2bdx)^2 \left( (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + bx)(c + dx) \right)^{1/3} \right)} \\
 & \quad \sqrt{\left( (bc - ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc - ad)^{2/3} \left( (a + bx)(c + dx) \right)^{1/3} + \right. \\
 & \quad \quad \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} \left( (a + bx)(c + dx) \right)^{2/3} \right) /} \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + bx)(c + dx) \right)^{1/3} \right)^2 \right) \text{EllipticF} \left[ \right. \\
 & \quad \left. \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + bx)(c + dx) \right)^{1/3}}{(1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + bx)(c + dx) \right)^{1/3}}, -7 - 4\sqrt{3} \right] \right] / \\
 & \quad \left( 2^{2/3} b^{1/3} (bc - ad) (a + bx)^{2/3} (c + dx)^{2/3} (bc + ad + 2bdx) \right. \\
 & \quad \sqrt{\left( (bc - ad)^{2/3} \left( (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + bx)(c + dx) \right)^{1/3} \right) \right) /} \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} \left( (a + bx)(c + dx) \right)^{1/3} \right)^2 \sqrt{(ad + b(c + 2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 83 leaves):

$$-\left( \left( 3 (c+dx)^{1/3} \left( 1 + \left( \frac{d(a+bx)}{-bc+ad} \right)^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad} \right] \right) \right) / \left( 2 (bc-ad) (a+bx)^{2/3} \right) \right)$$

**Problem 1613: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{8/3} (c+dx)^{2/3}} dx$$

Optimal (type 4, 621 leaves, 5 steps):

$$-\frac{3(c+dx)^{1/3}}{5(bc-ad)(a+bx)^{5/3}} + \frac{6d(c+dx)^{1/3}}{5(bc-ad)^2(a+bx)^{2/3}} + \left( 2 \times 2^{1/3} \times 3^{3/4} \sqrt{2+\sqrt{3}} d^{5/3} ((a+bx)(c+dx))^{2/3} \sqrt{(bc+ad+2bdx)^2} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right)} / \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}} \right], -7-4\sqrt{3} \right] \right) / \left( 5 b^{1/3} (bc-ad)^2 (a+bx)^{2/3} (c+dx)^{2/3} (bc+ad+2bdx) \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right)} / \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \sqrt{(ad+b(c+2dx))^2} \right)$$

Result (type 5, 102 leaves):

$$\left( 3 (c+dx)^{1/3} \left( -bc+3ad+2bdx+2d(a+bx) \left( \frac{d(a+bx)}{-bc+ad} \right)^{2/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad} \right] \right) \right) / \left( 5 (bc-ad)^2 (a+bx)^{5/3} \right)$$

**Problem 1614: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{11/3} (c+dx)^{2/3}} dx$$

Optimal (type 4, 656 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{3(c+dx)^{1/3}}{8(bc-ad)(a+bx)^{8/3}} + \frac{21d(c+dx)^{1/3}}{40(bc-ad)^2(a+bx)^{5/3}} - \\
 & \frac{21d^2(c+dx)^{1/3}}{20(bc-ad)^3(a+bx)^{2/3}} - \left( 7 \times 3^{3/4} \sqrt{2+\sqrt{3}} d^{8/3} ((a+bx)(c+dx))^{2/3} \right. \\
 & \sqrt{(bc+ad+2bdx)^2} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \\
 & \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + \right. \\
 & \quad \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) /} \\
 & \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \text{EllipticF}\left[ \right. \\
 & \quad \left. \text{ArcSin}\left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right] / \\
 & \left( 10 \times 2^{2/3} b^{1/3} (bc-ad)^3 (a+bx)^{2/3} (c+dx)^{2/3} (bc+ad+2bdx) \right. \\
 & \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) /} \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 136 leaves):

$$\begin{aligned}
 & \left( 3(c+dx)^{1/3} \right. \\
 & \left. \left( 26a^2d^2 + abd(-17c+35dx) + b^2(5c^2-7cdx+14d^2x^2) + 14d^2(a+bx)^2 \left( \frac{d(a+bx)}{-bc+ad} \right)^{2/3} \right. \right. \\
 & \quad \left. \left. \text{Hypergeometric2F1}\left[ \frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{b(c+dx)}{bc-ad} \right] \right) \right) / \left( 40(-bc+ad)^3(a+bx)^{8/3} \right)
 \end{aligned}$$

**Problem 1615: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{7/3}}{(c+dx)^{4/3}} dx$$

Optimal (type 3, 241 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{3(a+bx)^{7/3}}{d(c+dx)^{1/3}} - \frac{14b(bc-ad)(a+bx)^{1/3}(c+dx)^{2/3}}{3d^3} + \\
 & \frac{7b(a+bx)^{4/3}(c+dx)^{2/3}}{2d^2} - \frac{14b^{1/3}(bc-ad)^2 \text{ArcTan}\left[ \frac{1}{\sqrt{3}} + \frac{2b^{1/3}(c+dx)^{1/3}}{\sqrt{3}d^{1/3}(a+bx)^{1/3}} \right]}{3\sqrt{3}d^{10/3}} - \\
 & \frac{7b^{1/3}(bc-ad)^2 \text{Log}[a+bx]}{9d^{10/3}} - \frac{7b^{1/3}(bc-ad)^2 \text{Log}\left[-1 + \frac{b^{1/3}(c+dx)^{1/3}}{d^{1/3}(a+bx)^{1/3}}\right]}{3d^{10/3}}
 \end{aligned}$$

Result (type 5, 132 leaves):

$$\frac{1}{6 d^4 (a+b x)^{2/3}} (c+d x)^{2/3} \left( d (a+b x) \left( b (-10 b c+13 a d)+3 b^2 d x-\frac{18 (b c-a d)^2}{c+d x} \right) + 14 b (b c-a d)^2 \left( \frac{d (a+b x)}{-b c+a d} \right)^{2/3} \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c-a d}\right] \right)$$

**Problem 1616: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+b x)^{4/3}}{(c+d x)^{4/3}} dx$$

Optimal (type 3, 195 leaves, 3 steps):

$$-\frac{3 (a+b x)^{4/3}}{d (c+d x)^{1/3}} + \frac{4 b (a+b x)^{1/3} (c+d x)^{2/3}}{d^2} + \frac{4 b^{1/3} (b c-a d) \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{\sqrt{3} d^{7/3}} + \frac{2 b^{1/3} (b c-a d) \text{Log}[a+b x]}{3 d^{7/3}} + \frac{2 b^{1/3} (b c-a d) \text{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{d^{7/3}}$$

Result (type 5, 95 leaves):

$$\frac{(a+b x)^{1/3} (c+d x)^{2/3} \left( \frac{4 b c-3 a d+b d x}{c+d x} + \frac{2 b \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c-a d}\right]}{\left(\frac{d (a+b x)}{-b c+a d}\right)^{1/3}} \right)}{d^2}$$

**Problem 1617: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+b x)^{1/3}}{(c+d x)^{4/3}} dx$$

Optimal (type 3, 149 leaves, 2 steps):

$$-\frac{3 (a+b x)^{1/3}}{d (c+d x)^{1/3}} - \frac{\sqrt{3} b^{1/3} \text{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 b^{1/3} (c+d x)^{1/3}}{\sqrt{3} d^{1/3} (a+b x)^{1/3}}\right]}{d^{4/3}} - \frac{b^{1/3} \text{Log}[a+b x]}{2 d^{4/3}} - \frac{3 b^{1/3} \text{Log}\left[-1 + \frac{b^{1/3} (c+d x)^{1/3}}{d^{1/3} (a+b x)^{1/3}}\right]}{2 d^{4/3}}$$

Result (type 5, 90 leaves):

$$\left( -6 d (a+b x)+3 b \left( \frac{d (a+b x)}{-b c+a d} \right)^{2/3} (c+d x) \text{Hypergeometric2F1}\left[\frac{2}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c-a d}\right] \right) / \left( 2 d^2 (a+b x)^{2/3} (c+d x)^{1/3} \right)$$



Problem 1622: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{8/3}}{(c + d x)^{4/3}} dx$$

Optimal (type 4, 1355 leaves, 8 steps):

$$\begin{aligned}
 & -\frac{3(a+bx)^{8/3}}{d(c+dx)^{1/3}} - \frac{30b(bc-ad)(a+bx)^{2/3}(c+dx)^{2/3}}{7d^3} + \frac{24b(a+bx)^{5/3}(c+dx)^{2/3}}{7d^2} + \\
 & \left( 30 \times 2^{2/3} b^{1/3} (bc-ad)^2 ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2} \right) / \\
 & \left( 7d^{11/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\
 & \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) - \\
 & \left( 15 \times 2^{2/3} \times 3^{1/4} \sqrt{2-\sqrt{3}} b^{1/3} (bc-ad)^{8/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} \right. \\
 & \left. (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \\
 & \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right] \text{EllipticE} \left[ \right. \\
 & \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right] / \\
 & \left( 7d^{11/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\
 & \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) / \right. \\
 & \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \sqrt{(ad+b(c+2dx))^2} + \\
 & \left( 20 \times 2^{1/6} \times 3^{3/4} b^{1/3} (bc-ad)^{8/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \\
 & \left. \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + \right. \right. \\
 & \left. \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \right. \\
 & \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right] \text{EllipticF} \left[ \right. \\
 & \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right] / \\
 & \left( 7d^{11/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\
 & \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) / \right. \\
 & \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \sqrt{(ad+b(c+2dx))^2}
 \end{aligned}$$

Result (type 5, 131 leaves):

$$\frac{1}{7 d^4 (a+b x)^{1/3}} 3 (c+d x)^{2/3} \left( d (a+b x) \left( b (-3 b c+4 a d) + b^2 d x - \frac{7 (b c-a d)^2}{c+d x} \right) + 10 b (b c-a d)^2 \left( \frac{d (a+b x)}{-b c+a d} \right)^{1/3} \text{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c+d x)}{b c-a d} \right] \right)$$

**Problem 1623: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+b x)^{5/3}}{(c+d x)^{4/3}} dx$$

Optimal (type 4, 1317 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{3(a+bx)^{5/3}}{d(c+dx)^{1/3}} + \frac{15b(a+bx)^{2/3}(c+dx)^{2/3}}{4d^2} - \\
 & \left( \frac{15b^{1/3}(bc-ad)((a+bx)(c+dx))^{1/3}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2}}{2 \times 2^{1/3}d^{8/3}(a+bx)^{1/3}(c+dx)^{1/3}(bc+ad+2bdx)} \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3}b^{1/3}d^{1/3}((a+bx)(c+dx))^{1/3} \right) + \right. \\
 & \left( 15 \times 3^{1/4}\sqrt{2-\sqrt{3}}b^{1/3}(bc-ad)^{5/3}((a+bx)(c+dx))^{1/3}\sqrt{(bc+ad+2bdx)^2} \right. \\
 & \quad \left. \left( (bc-ad)^{2/3} + 2^{2/3}b^{1/3}d^{1/3}((a+bx)(c+dx))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3}b^{1/3}d^{1/3} \right. \right. \\
 & \quad \quad \left. \left. (bc-ad)^{2/3}((a+bx)(c+dx))^{1/3} + 2 \times 2^{1/3}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3} \right)} \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3}b^{1/3}d^{1/3}((a+bx)(c+dx))^{1/3} \right)^2 \right) \text{EllipticE}\left[ \right. \\
 & \quad \left. \text{ArcSin}\left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3}b^{1/3}d^{1/3}((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3}b^{1/3}d^{1/3}((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right] \left. \right) / \\
 & \left( 4 \times 2^{1/3}d^{8/3}(a+bx)^{1/3}(c+dx)^{1/3}(bc+ad+2bdx) \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3}b^{1/3}d^{1/3}((a+bx)(c+dx))^{1/3} \right) \right)} \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3}b^{1/3}d^{1/3}((a+bx)(c+dx))^{1/3} \right)^2 \right) \sqrt{(ad+b(c+2dx))^2} - \\
 & \left( 5 \times 3^{3/4}b^{1/3}(bc-ad)^{5/3}((a+bx)(c+dx))^{1/3}\sqrt{(bc+ad+2bdx)^2} \right. \\
 & \quad \left. \left( (bc-ad)^{2/3} + 2^{2/3}b^{1/3}d^{1/3}((a+bx)(c+dx))^{1/3} \right) \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3}b^{1/3}d^{1/3}(bc-ad)^{2/3}((a+bx)(c+dx))^{1/3} + \right. \right. \\
 & \quad \quad \left. \left. 2 \times 2^{1/3}b^{2/3}d^{2/3}((a+bx)(c+dx))^{2/3} \right)} \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3}b^{1/3}d^{1/3}((a+bx)(c+dx))^{1/3} \right)^2 \right) \text{EllipticF}\left[ \right. \\
 & \quad \left. \text{ArcSin}\left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3}b^{1/3}d^{1/3}((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3}b^{1/3}d^{1/3}((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right] \left. \right) / \\
 & \left( 2^{5/6}d^{8/3}(a+bx)^{1/3}(c+dx)^{1/3}(bc+ad+2bdx) \right. \\
 & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3}b^{1/3}d^{1/3}((a+bx)(c+dx))^{1/3} \right) \right)} \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3}b^{1/3}d^{1/3}((a+bx)(c+dx))^{1/3} \right)^2 \right) \sqrt{(ad+b(c+2dx))^2}
 \end{aligned}$$

Result (type 5, 98 leaves):

$$\frac{1}{4d^2} 3(a+bx)^{2/3}(c+dx)^{2/3} \left( \frac{5bc-4ad+bdx}{c+dx} + \frac{5b \text{Hypergeometric2F1}\left[\frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b(c+dx)}{bc-ad}\right]}{\left(\frac{d(a+bx)}{-bc+ad}\right)^{2/3}} \right)$$

**Problem 1624: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{2/3}}{(c+dx)^{4/3}} dx$$

Optimal (type 4, 1279 leaves, 6 steps):

$$\begin{aligned} & -\frac{3(a+bx)^{2/3}}{d(c+dx)^{1/3}} + \left( 3 \times 2^{2/3} b^{1/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2} \right) / \\ & \left( d^{5/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\ & \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) - \right. \\ & \left. \left( 3 \times 3^{1/4} \sqrt{2-\sqrt{3}} b^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \right. \\ & \left. \left. \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} \right. \right. \right. \\ & \left. \left. \left. (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \right. \right. \\ & \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \text{EllipticE} \left[ \right. \\ & \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right) / \\ & \left( 2^{1/3} d^{5/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\ & \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) / \right. \right. \\ & \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \sqrt{(ad+b(c+2dx))^2} \right) + \\ & \left( 2 \times 2^{1/6} \times 3^{3/4} b^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \\ & \left. \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right. \\ & \left. \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + \right. \right. \\ & \left. \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \right. \\ & \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \text{EllipticF} \left[ \right. \\ & \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right) / \\ & \left( d^{5/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\ & \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) / \right. \right. \\ & \left. \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \sqrt{(ad+b(c+2dx))^2} \right) \end{aligned}$$

Result (type 5, 87 leaves):

$$\left( -3 d (a + b x) + 3 b \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} (c + d x) \operatorname{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right] \right) / \left( d^2 (a + b x)^{1/3} (c + d x)^{1/3} \right)$$

**Problem 1625: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b x)^{1/3} (c + d x)^{4/3}} dx$$

Optimal (type 4, 1298 leaves, 6 steps):

$$\begin{aligned}
 & \frac{3 (a+bx)^{2/3}}{(bc-ad)(c+dx)^{1/3}} - \\
 & \left( 3b^{1/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2} \right) / \\
 & \left( 2^{1/3} d^{2/3} (bc-ad)(a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) + \\
 & \left( 3 \times 3^{1/4} \sqrt{2-\sqrt{3}} b^{1/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \quad \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} \right.} \\
 & \quad \left. (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \\
 & \quad \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \text{EllipticE}\left[ \right. \\
 & \quad \left. \text{ArcSin}\left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right] / \\
 & \left( 2 \times 2^{1/3} d^{2/3} (bc-ad)^{1/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\
 & \quad \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) /} \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \sqrt{(ad+b(c+2dx))^2} \right) - \\
 & \left( 2^{1/6} \times 3^{3/4} b^{1/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \quad \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \\
 & \quad \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + \right.} \\
 & \quad \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) /} \\
 & \quad \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \text{EllipticF}\left[ \right. \\
 & \quad \left. \text{ArcSin}\left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}, -7-4\sqrt{3} \right] \right] / \\
 & \left( d^{2/3} (bc-ad)^{1/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\
 & \quad \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) /} \\
 & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 100 leaves):

$$\begin{aligned}
 & \left( 6d(a+bx) - 3b \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/3} (c+dx) \text{Hypergeometric2F1}\left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b(c+dx)}{bc-ad} \right] \right) / \\
 & \left( 2d(bc-ad)(a+bx)^{1/3}(c+dx)^{1/3} \right)
 \end{aligned}$$

### Problem 1626: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+bx)^{4/3} (c+dx)^{4/3}} dx$$

Optimal (type 4, 1327 leaves, 7 steps):

$$\begin{aligned} & -\frac{3}{(bc-ad)(a+bx)^{1/3}(c+dx)^{1/3}} - \frac{6d(a+bx)^{2/3}}{(bc-ad)^2(c+dx)^{1/3}} + \\ & \left( 3 \times 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2} \right) / \\ & \left( (bc-ad)^2 (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\ & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) - \\ & \left( 3 \times 3^{1/4} \sqrt{2-\sqrt{3}} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \\ & \quad \left. \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} \right. \right. \\ & \quad \left. \left. (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \right. \\ & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \text{EllipticE} \left[ \right. \\ & \quad \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}} \right], -7-4\sqrt{3} \right] \left. \right) / \\ & \left( 2^{1/3} (bc-ad)^{4/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\ & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) / \right. \\ & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \sqrt{(ad+b(c+2dx))^2} + \\ & \left( 2 \times 2^{1/6} \times 3^{3/4} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \sqrt{(bc+ad+2bdx)^2} \right. \\ & \quad \left. \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right. \\ & \quad \left. \sqrt{\left( (bc-ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc-ad)^{2/3} ((a+bx)(c+dx))^{1/3} + \right. \right. \\ & \quad \left. \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a+bx)(c+dx))^{2/3} \right) / \right. \\ & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \text{EllipticF} \left[ \right. \\ & \quad \left. \text{ArcSin} \left[ \frac{(1-\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}}{(1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3}} \right], -7-4\sqrt{3} \right] \left. \right) / \\ & \left( (bc-ad)^{4/3} (a+bx)^{1/3} (c+dx)^{1/3} (bc+ad+2bdx) \right. \\ & \quad \left. \sqrt{\left( (bc-ad)^{2/3} \left( (bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right) \right) / \right. \\ & \quad \left. \left( (1+\sqrt{3})(bc-ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a+bx)(c+dx))^{1/3} \right)^2 \right) \sqrt{(ad+b(c+2dx))^2} \end{aligned}$$



Result (type 5, 98 leaves):

$$- \left( \left( 3 \left( a d + b (c + 2 d x) - b \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} (c + d x) \operatorname{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right] \right) \right) / \left( (b c - a d)^2 (a + b x)^{1/3} (c + d x)^{1/3} \right) \right)$$

Problem 1627: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{7/3} (c + d x)^{4/3}} dx$$

Optimal (type 4, 1370 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{3}{4 (bc - ad) (a + bx)^{4/3} (c + dx)^{1/3}} + \\
 & \frac{15d}{4 (bc - ad)^2 (a + bx)^{1/3} (c + dx)^{1/3}} + \frac{15d^2 (a + bx)^{2/3}}{2 (bc - ad)^3 (c + dx)^{1/3}} - \\
 & \left( \frac{15 b^{1/3} d^{4/3} ((a + bx) (c + dx))^{1/3} \sqrt{(bc + ad + 2bdx)^2} \sqrt{(ad + b(c + 2dx))^2}}{2 \times 2^{1/3} (bc - ad)^3 (a + bx)^{1/3} (c + dx)^{1/3} (bc + ad + 2bdx)} \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) + \right. \\
 & \left. \left( 15 \times 3^{1/4} \sqrt{2 - \sqrt{3}} b^{1/3} d^{4/3} ((a + bx) (c + dx))^{1/3} \sqrt{(bc + ad + 2bdx)^2} \right. \right. \\
 & \quad \left. \left( (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) \sqrt{\left( (bc - ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} \right. \right. \\
 & \quad \left. \left. (bc - ad)^{2/3} ((a + bx) (c + dx))^{1/3} + 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + bx) (c + dx))^{2/3} \right)} \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right)^2 \right) \text{EllipticE} \left[ \right. \\
 & \quad \left. \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3}}{(1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3}}, -7 - 4\sqrt{3} \right] \right] \Bigg/ \\
 & \left( 4 \times 2^{1/3} (bc - ad)^{7/3} (a + bx)^{1/3} (c + dx)^{1/3} (bc + ad + 2bdx) \right. \\
 & \quad \left. \sqrt{\left( (bc - ad)^{2/3} \left( (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) \right)} \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right)^2 \sqrt{(ad + b(c + 2dx))^2} \right) - \\
 & \left( 5 \times 3^{3/4} b^{1/3} d^{4/3} ((a + bx) (c + dx))^{1/3} \sqrt{(bc + ad + 2bdx)^2} \right. \\
 & \quad \left( (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) \\
 & \quad \left. \sqrt{\left( (bc - ad)^{4/3} - 2^{2/3} b^{1/3} d^{1/3} (bc - ad)^{2/3} ((a + bx) (c + dx))^{1/3} + \right. \right. \\
 & \quad \left. \left. 2 \times 2^{1/3} b^{2/3} d^{2/3} ((a + bx) (c + dx))^{2/3} \right)} \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right)^2 \right) \text{EllipticF} \left[ \right. \\
 & \quad \left. \text{ArcSin} \left[ \frac{(1 - \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3}}{(1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3}}, -7 - 4\sqrt{3} \right] \right] \Bigg/ \\
 & \left( 2^{5/6} (bc - ad)^{7/3} (a + bx)^{1/3} (c + dx)^{1/3} (bc + ad + 2bdx) \right. \\
 & \quad \left. \sqrt{\left( (bc - ad)^{2/3} \left( (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right) \right)} \right. \\
 & \quad \left. \left( (1 + \sqrt{3}) (bc - ad)^{2/3} + 2^{2/3} b^{1/3} d^{1/3} ((a + bx) (c + dx))^{1/3} \right)^2 \sqrt{(ad + b(c + 2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 138 leaves):

$$\begin{aligned}
 & - \left( \left( 3 \left( 4 a^2 d^2 + a b d (7 c + 15 d x) + b^2 (-c^2 + 5 c d x + 10 d^2 x^2) - \right. \right. \right. \\
 & \quad \left. \left. \left. 5 b d (a + b x) \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/3} (c + d x) \operatorname{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{2}{3}, \frac{5}{3}, \frac{b (c + d x)}{b c - a d} \right] \right) \right) / \\
 & \quad \left( 4 (-b c + a d)^3 (a + b x)^{4/3} (c + d x)^{1/3} \right)
 \end{aligned}$$

**Problem 1628: Result unnecessarily involves higher level functions.**

$$\int \frac{(-1+x)^{1/3}}{(1+x)^{1/3}} dx$$

Optimal (type 3, 77 leaves, 2 steps):

$$(-1+x)^{1/3} (1+x)^{2/3} + \frac{2 \operatorname{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2(1+x)^{1/3}}{\sqrt{3}(-1+x)^{1/3}} \right]}{\sqrt{3}} + \frac{1}{3} \operatorname{Log}[-1+x] + \operatorname{Log} \left[ -1 + \frac{(1+x)^{1/3}}{(-1+x)^{1/3}} \right]$$

Result (type 5, 50 leaves):

$$\left( \frac{-1+x}{1+x} \right)^{1/3} \left( 1+x - 2^{2/3} (1+x)^{1/3} \operatorname{Hypergeometric2F1} \left[ \frac{1}{3}, \frac{1}{3}, \frac{4}{3}, \frac{1-x}{2} \right] \right)$$

**Problem 1629: Result unnecessarily involves higher level functions.**

$$\int (a+bx)^{3/2} (c+dx)^{1/4} dx$$

Optimal (type 4, 185 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{8 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{1/4}}{77 b d^2} + \frac{4 (b c - a d) (a + b x)^{3/2} (c + d x)^{1/4}}{77 b d} + \\
 & \frac{4 (a + b x)^{5/2} (c + d x)^{1/4}}{11 b} + \frac{16 (b c - a d)^{13/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{b^{1/4} (c+dx)^{1/4}}{(bc-ad)^{1/4}} \right], -1 \right]}{77 b^{5/4} d^3 \sqrt{a + b x}}
 \end{aligned}$$

Result (type 5, 140 leaves):

$$\begin{aligned}
 & - \frac{1}{77 b d^3 \sqrt{a + b x}} 4 (c + d x)^{1/4} \left( -d (a + b x) (4 a^2 d^2 + a b d (5 c + 13 d x) + b^2 (-2 c^2 + c d x + 7 d^2 x^2)) - \right. \\
 & \quad \left. 4 (b c - a d)^3 \sqrt{\frac{d (a + b x)}{-b c + a d}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d} \right] \right)
 \end{aligned}$$

### Problem 1630: Result unnecessarily involves higher level functions.

$$\int \sqrt{a+bx} (c+dx)^{1/4} dx$$

Optimal (type 4, 147 leaves, 5 steps):

$$\frac{4 (bc-ad) \sqrt{a+bx} (c+dx)^{1/4}}{21bd} + \frac{4 (a+bx)^{3/2} (c+dx)^{1/4}}{7b} - \frac{8 (bc-ad)^{9/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{21b^{5/4}d^2\sqrt{a+bx}}$$

Result (type 5, 109 leaves):

$$\frac{1}{21bd^2\sqrt{a+bx}} (c+dx)^{1/4} \left( d(a+bx)(2ad+b(c+3dx)) - 2(bc-ad)^2 \sqrt{\frac{d(a+bx)}{-bc+ad}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

### Problem 1631: Result unnecessarily involves higher level functions.

$$\int \frac{(c+dx)^{1/4}}{\sqrt{a+bx}} dx$$

Optimal (type 4, 111 leaves, 4 steps):

$$\frac{4\sqrt{a+bx}(c+dx)^{1/4}}{3b} + \frac{4(bc-ad)^{5/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{3b^{5/4}d\sqrt{a+bx}}$$

Result (type 5, 93 leaves):

$$\frac{1}{3bd\sqrt{a+bx}} (c+dx)^{1/4} \left( d(a+bx) + (bc-ad) \sqrt{\frac{d(a+bx)}{-bc+ad}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

### Problem 1632: Result unnecessarily involves higher level functions.

$$\int \frac{(c+dx)^{1/4}}{(a+bx)^{3/2}} dx$$

Optimal (type 4, 104 leaves, 4 steps):

$$-\frac{2(c+dx)^{1/4}}{b\sqrt{a+bx}} + \frac{2(bc-ad)^{1/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{b^{5/4}\sqrt{a+bx}}$$

Result (type 5, 74 leaves):

$$\frac{2(c+dx)^{1/4} \left(-1 + \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad}\right]\right)}{b\sqrt{a+bx}}$$

**Problem 1633: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{1/4}}{(a+bx)^{5/2}} dx$$

Optimal (type 4, 145 leaves, 5 steps):

$$-\frac{2(c+dx)^{1/4}}{3b(a+bx)^{3/2}} - \frac{d(c+dx)^{1/4}}{3b(bc-ad)\sqrt{a+bx}} - \frac{d \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{3b^{5/4}(bc-ad)^{3/4}\sqrt{a+bx}}$$

Result (type 5, 103 leaves):

$$\left( (c+dx)^{1/4} \left( 2bc-ad+bdx+d(a+bx) \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) / (3b(-bc+ad)(a+bx)^{3/2})$$

**Problem 1634: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{1/4}}{(a+bx)^{7/2}} dx$$

Optimal (type 4, 185 leaves, 6 steps):

$$-\frac{2(c+dx)^{1/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{1/4}}{15b(bc-ad)(a+bx)^{3/2}} + \frac{d^2(c+dx)^{1/4}}{6b(bc-ad)^2\sqrt{a+bx}} + \frac{d^2 \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{6b^{5/4}(bc-ad)^{7/4}\sqrt{a+bx}}$$

Result (type 5, 140 leaves):

$$\left( (c+dx)^{1/4} \left( -5a^2d^2 + 2abd(11c+6dx) + b^2(-12c^2 - 2cdx + 5d^2x^2) + 5d^2(a+bx)^2 \right. \right. \\ \left. \left. \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) / (30b(bc-ad)^2(a+bx)^{5/2})$$

**Problem 1635: Result unnecessarily involves higher level functions.**

$$\int (a+bx)^{3/2} (c+dx)^{3/4} dx$$

Optimal (type 4, 270 leaves, 10 steps):

$$-\frac{8(bc-ad)^2\sqrt{a+bx}(c+dx)^{3/4}}{65bd^2} + \frac{4(bc-ad)(a+bx)^{3/2}(c+dx)^{3/4}}{39bd} + \\ \frac{4(a+bx)^{5/2}(c+dx)^{3/4}}{13b} + \frac{16(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{65b^{7/4}d^3\sqrt{a+bx}} - \\ \frac{16(bc-ad)^{15/4}\sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{65b^{7/4}d^3\sqrt{a+bx}}$$

Result (type 5, 141 leaves):

$$-\frac{1}{195bd^3\sqrt{a+bx}} \\ 4(c+dx)^{3/4} \left( -d(a+bx)(4a^2d^2 + abd(17c+25dx) + b^2(-6c^2 + 5cdx + 15d^2x^2)) - \right. \\ \left. 4(bc-ad)^3 \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

**Problem 1636: Result unnecessarily involves higher level functions.**

$$\int \sqrt{a+bx} (c+dx)^{3/4} dx$$

Optimal (type 4, 232 leaves, 9 steps):

$$\frac{4 (b c - a d) \sqrt{a + b x} (c + d x)^{3/4}}{15 b d} + \frac{4 (a + b x)^{3/2} (c + d x)^{3/4}}{9 b} -$$

$$\frac{8 (b c - a d)^{11/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{15 b^{7/4} d^2 \sqrt{a + b x}} +$$

$$\frac{8 (b c - a d)^{11/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{15 b^{7/4} d^2 \sqrt{a + b x}}$$

Result (type 5, 110 leaves):

$$\frac{1}{45 b d^2 \sqrt{a + b x}} 4 (c + d x)^{3/4} \left( d (a + b x) (3 b c + 2 a d + 5 b d x) - \right.$$

$$\left. 2 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

**Problem 1637: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + d x)^{3/4}}{\sqrt{a + b x}} dx$$

Optimal (type 4, 196 leaves, 8 steps):

$$\frac{4 \sqrt{a + b x} (c + d x)^{3/4}}{5 b} + \frac{12 (b c - a d)^{7/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{5 b^{7/4} d \sqrt{a + b x}} -$$

$$\frac{12 (b c - a d)^{7/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{5 b^{7/4} d \sqrt{a + b x}}$$

Result (type 5, 93 leaves):

$$\frac{1}{5 b d \sqrt{a + b x}}$$

$$4 (c + d x)^{3/4} \left( d (a + b x) + (b c - a d) \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

**Problem 1638: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{3/4}}{(a+bx)^{3/2}} dx$$

Optimal (type 4, 184 leaves, 8 steps):

$$\frac{2(c+dx)^{3/4}}{b\sqrt{a+bx}} + \frac{6(bc-ad)^{3/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{b^{7/4}\sqrt{a+bx}} - \frac{6(bc-ad)^{3/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{b^{7/4}\sqrt{a+bx}}$$

Result (type 5, 74 leaves):

$$\frac{2(c+dx)^{3/4} \left(-1 + \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right]\right)}{b\sqrt{a+bx}}$$

**Problem 1639: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{3/4}}{(a+bx)^{5/2}} dx$$

Optimal (type 4, 221 leaves, 9 steps):

$$\frac{2(c+dx)^{3/4}}{3b(a+bx)^{3/2}} - \frac{d(c+dx)^{3/4}}{b(bc-ad)\sqrt{a+bx}} + \frac{d \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{b^{7/4}(bc-ad)^{1/4}\sqrt{a+bx}} - \frac{d \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{b^{7/4}(bc-ad)^{1/4}\sqrt{a+bx}}$$

Result (type 5, 104 leaves):

$$\left( (c+dx)^{3/4} \left( \frac{2bc+ad+3bdx-d(a+bx) \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right]}{3b(-bc+ad)(a+bx)^{3/2}} \right) \right) /$$



**Problem 1640: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{3/4}}{(a+bx)^{7/2}} dx$$

Optimal (type 4, 270 leaves, 10 steps):

$$\begin{aligned} & -\frac{2(c+dx)^{3/4}}{5b(a+bx)^{5/2}} - \frac{d(c+dx)^{3/4}}{5b(bc-ad)(a+bx)^{3/2}} + \\ & \frac{3d^2(c+dx)^{3/4}}{10b(bc-ad)^2\sqrt{a+bx}} - \frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{10b^{7/4}(bc-ad)^{5/4}\sqrt{a+bx}} + \\ & \frac{3d^2\sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{10b^{7/4}(bc-ad)^{5/4}\sqrt{a+bx}} \end{aligned}$$

Result (type 5, 140 leaves):

$$\begin{aligned} & \left( (c+dx)^{3/4} \left( a^2 d^2 + 2abd(3c+4dx) - b^2(4c^2 + 2cdx - 3d^2x^2) - d^2(a+bx)^2 \sqrt{\frac{d(a+bx)}{-bc+ad}} \right. \right. \\ & \left. \left. \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) / \left( 10b(bc-ad)^2(a+bx)^{5/2} \right) \end{aligned}$$

**Problem 1641: Result unnecessarily involves higher level functions.**

$$\int (a+bx)^{3/2} (c+dx)^{5/4} dx$$

Optimal (type 4, 220 leaves, 7 steps):

$$\begin{aligned} & -\frac{8(bc-ad)^3\sqrt{a+bx}(c+dx)^{1/4}}{231b^2d^2} + \frac{4(bc-ad)^2(a+bx)^{3/2}(c+dx)^{1/4}}{231b^2d} + \\ & \frac{4(bc-ad)(a+bx)^{5/2}(c+dx)^{1/4}}{33b^2} + \frac{4(a+bx)^{5/2}(c+dx)^{5/4}}{15b} + \\ & \frac{16(bc-ad)^{17/4}\sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{231b^{9/4}d^3\sqrt{a+bx}} \end{aligned}$$

Result (type 5, 182 leaves):

$$\left( 4 (c + dx)^{1/4} \left( -d (a + bx) (20 a^3 d^3 - 12 a^2 b d^2 (6c + dx) - a b^2 d (35 c^2 + 214 c dx + 119 d^2 x^2)) + \right. \right. \\ \left. \left. b^3 (10 c^3 - 5 c^2 dx - 112 c d^2 x^2 - 77 d^3 x^3) \right) + 20 (bc - ad)^4 \sqrt{\frac{d(a + bx)}{-bc + ad}} \right. \\ \left. \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c + dx)}{bc - ad} \right] \right) / (1155 b^2 d^3 \sqrt{a + bx})$$

**Problem 1642: Result unnecessarily involves higher level functions.**

$$\int \sqrt{a + bx} (c + dx)^{5/4} dx$$

Optimal (type 4, 182 leaves, 6 steps):

$$\frac{20 (bc - ad)^2 \sqrt{a + bx} (c + dx)^{1/4}}{231 b^2 d} + \frac{20 (bc - ad) (a + bx)^{3/2} (c + dx)^{1/4}}{77 b^2} + \\ \frac{4 (a + bx)^{3/2} (c + dx)^{5/4}}{11 b} - \frac{40 (bc - ad)^{13/4} \sqrt{-\frac{d(a + bx)}{bc - ad}} \text{EllipticF} \left[ \text{ArcSin} \left[ \frac{b^{1/4} (c + dx)^{1/4}}{(bc - ad)^{1/4}} \right], -1 \right]}{231 b^{9/4} d^2 \sqrt{a + bx}}$$

Result (type 5, 143 leaves):

$$\frac{1}{231 b^2 d^2 \sqrt{a + bx}} \\ 4 (c + dx)^{1/4} \left( -d (a + bx) (10 a^2 d^2 - 2 a b d (13 c + 3 dx) - b^2 (5 c^2 + 36 c dx + 21 d^2 x^2)) - \right. \\ \left. 10 (bc - ad)^3 \sqrt{\frac{d(a + bx)}{-bc + ad}} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c + dx)}{bc - ad} \right] \right)$$

**Problem 1643: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + dx)^{5/4}}{\sqrt{a + bx}} dx$$

Optimal (type 4, 144 leaves, 5 steps):

$$\frac{20 (b c - a d) \sqrt{a + b x} (c + d x)^{1/4}}{21 b^2} + \frac{4 \sqrt{a + b x} (c + d x)^{5/4}}{7 b} +$$

$$\frac{20 (b c - a d)^{9/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{21 b^{9/4} d \sqrt{a + b x}}$$

Result (type 5, 111 leaves):

$$\frac{1}{21 b^2 d \sqrt{a + b x}} 4 (c + d x)^{1/4} \left( -d (a + b x) (-8 b c + 5 a d - 3 b d x) + \right.$$

$$\left. 5 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

**Problem 1644: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + d x)^{5/4}}{(a + b x)^{3/2}} dx$$

Optimal (type 4, 132 leaves, 5 steps):

$$\frac{10 d \sqrt{a + b x} (c + d x)^{1/4}}{3 b^2} - \frac{2 (c + d x)^{5/4}}{b \sqrt{a + b x}} +$$

$$\frac{10 (b c - a d)^{5/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{3 b^{9/4} \sqrt{a + b x}}$$

Result (type 5, 95 leaves):

$$-\frac{1}{3 b^2 \sqrt{a + b x}}$$

$$2 (c + d x)^{1/4} \left( 3 b c - 5 a d - 2 b d x + \frac{5 d (a + b x) \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right]}{\sqrt{\frac{d (a + b x)}{-b c + a d}}} \right)$$

**Problem 1645: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + d x)^{5/4}}{(a + b x)^{5/2}} dx$$

Optimal (type 4, 135 leaves, 5 steps):

$$-\frac{5d(c+dx)^{1/4}}{3b^2\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{3b(a+bx)^{3/2}} + \frac{5d(bc-ad)^{1/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{3b^{9/4}\sqrt{a+bx}}$$

Result (type 5, 95 leaves):

$$\frac{1}{3b^2(a+bx)^{3/2}}(c+dx)^{1/4} \left( -2bc - 5ad - 7bdx + 5d(a+bx) \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

**Problem 1646: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{7/2}} dx$$

Optimal (type 4, 175 leaves, 6 steps):

$$-\frac{d(c+dx)^{1/4}}{3b^2(a+bx)^{3/2}} - \frac{d^2(c+dx)^{1/4}}{6b^2(bc-ad)\sqrt{a+bx}} - \frac{2(c+dx)^{5/4}}{5b(a+bx)^{5/2}} - \frac{d^2 \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{6b^{9/4}(bc-ad)^{3/4}\sqrt{a+bx}}$$

Result (type 5, 138 leaves):

$$\left( (c+dx)^{1/4} \left( -5a^2d^2 - 2abd(c+6dx) + b^2(12c^2 + 22cdx + 5d^2x^2) + 5d^2(a+bx)^2 \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) / (30b^2(-bc+ad)(a+bx)^{5/2})$$

**Problem 1647: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/2}} dx$$

Optimal (type 4, 213 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{d (c+dx)^{1/4}}{7b^2 (a+bx)^{5/2}} - \frac{d^2 (c+dx)^{1/4}}{42b^2 (bc-ad) (a+bx)^{3/2}} + \frac{5d^3 (c+dx)^{1/4}}{84b^2 (bc-ad)^2 \sqrt{a+bx}} - \\
 & \frac{2 (c+dx)^{5/4}}{7b (a+bx)^{7/2}} + \frac{5d^3 \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{84b^{9/4} (bc-ad)^{7/4} \sqrt{a+bx}}
 \end{aligned}$$

Result (type 5, 181 leaves):

$$\begin{aligned}
 & \left( (c+dx)^{1/4} \left( -5a^3d^3 - a^2bd^2(2c+17dx) + ab^2d(36c^2+68cdx+17d^2x^2) - \right. \right. \\
 & \quad \left. \left. b^3(24c^3+36c^2dx+2cd^2x^2-5d^3x^3) + 5d^3(a+bx)^3 \sqrt{\frac{d(a+bx)}{-bc+ad}} \right. \right. \\
 & \quad \left. \left. \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) / \left( 84b^2 (bc-ad)^2 (a+bx)^{7/2} \right)
 \end{aligned}$$

**Problem 1648: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{1/4}} dx$$

Optimal (type 4, 264 leaves, 10 steps):

$$\begin{aligned}
 & \frac{16 (bc-ad)^2 \sqrt{a+bx} (c+dx)^{3/4}}{39d^3} - \frac{40 (bc-ad) (a+bx)^{3/2} (c+dx)^{3/4}}{117d^2} + \\
 & \frac{4 (a+bx)^{5/2} (c+dx)^{3/4}}{13d} - \frac{32 (bc-ad)^{15/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{39b^{3/4} d^4 \sqrt{a+bx}} + \\
 & \frac{32 (bc-ad)^{15/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{39b^{3/4} d^4 \sqrt{a+bx}}
 \end{aligned}$$

Result (type 5, 138 leaves):

$$\frac{1}{117 d^4 \sqrt{a+bx}} - 4 (c+dx)^{3/4} \left( d (a+bx) (31 a^2 d^2 + 2 a b d (-17 c + 14 d x) + b^2 (12 c^2 - 10 c d x + 9 d^2 x^2)) - 8 (bc-ad)^3 \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

**Problem 1649: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{1/4}} dx$$

Optimal (type 4, 229 leaves, 9 steps):

$$-\frac{8 (bc-ad) \sqrt{a+bx} (c+dx)^{3/4}}{15 d^2} + \frac{4 (a+bx)^{3/2} (c+dx)^{3/4}}{9 d} + \frac{16 (bc-ad)^{11/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{15 b^{3/4} d^3 \sqrt{a+bx}} - \frac{16 (bc-ad)^{11/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{15 b^{3/4} d^3 \sqrt{a+bx}}$$

Result (type 5, 107 leaves):

$$\frac{1}{45 d^3 \sqrt{a+bx}} 4 (c+dx)^{3/4} \left( d (a+bx) (-6 bc + 11 a d + 5 b d x) + 4 (bc-ad)^2 \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

**Problem 1650: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{1/4}} dx$$

Optimal (type 4, 196 leaves, 8 steps):

$$\frac{4 \sqrt{a+bx} (c+dx)^{3/4}}{5d} - \frac{8 (bc-ad)^{7/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{5b^{3/4}d^2\sqrt{a+bx}} +$$

$$\frac{8 (bc-ad)^{7/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{5b^{3/4}d^2\sqrt{a+bx}}$$

Result (type 5, 77 leaves):

$$\frac{4 \sqrt{a+bx} (c+dx)^{3/4} \left( 3 + \frac{{}_2\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right]}{\sqrt{\frac{d(a+bx)}{-bc+ad}}}\right)}{15d}$$

**Problem 1651: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{a+bx} (c+dx)^{1/4}} dx$$

Optimal (type 4, 167 leaves, 7 steps):

$$\frac{4 (bc-ad)^{3/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{b^{3/4}d\sqrt{a+bx}} -$$

$$\frac{4 (bc-ad)^{3/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{b^{3/4}d\sqrt{a+bx}}$$

Result (type 5, 73 leaves):

$$\frac{4 \sqrt{\frac{d(a+bx)}{-bc+ad}} (c+dx)^{3/4} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right]}{3d\sqrt{a+bx}}$$

**Problem 1652: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{3/2} (c+dx)^{1/4}} dx$$

Optimal (type 4, 191 leaves, 8 steps):

$$-\frac{2(c+dx)^{3/4}}{(bc-ad)\sqrt{a+bx}} + \frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{b^{3/4}(bc-ad)^{1/4}\sqrt{a+bx}} -$$

$$\frac{2\sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{b^{3/4}(bc-ad)^{1/4}\sqrt{a+bx}}$$

Result (type 5, 83 leaves):

$$\frac{2(c+dx)^{3/4} \left( -3 + \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)}{3(bc-ad)\sqrt{a+bx}}$$

Problem 1653: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{1/4}} dx$$

Optimal (type 4, 224 leaves, 9 steps):

$$-\frac{2(c+dx)^{3/4}}{3(bc-ad)(a+bx)^{3/2}} + \frac{d(c+dx)^{3/4}}{(bc-ad)^2\sqrt{a+bx}} -$$

$$\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{b^{3/4}(bc-ad)^{5/4}\sqrt{a+bx}} +$$

$$\frac{d\sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{b^{3/4}(bc-ad)^{5/4}\sqrt{a+bx}}$$

Result (type 5, 102 leaves):

$$\left( (c+dx)^{3/4} \right.$$

$$\left. \left( -2bc + 5ad + 3bdx - d(a+bx) \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) /$$

$$(3(bc-ad)^2(a+bx)^{3/2})$$



**Problem 1654: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{3/4}} dx$$

Optimal (type 4, 144 leaves, 5 steps):

$$-\frac{8(bc-ad)\sqrt{a+bx}(c+dx)^{1/4}}{7d^2} + \frac{4(a+bx)^{3/2}(c+dx)^{1/4}}{7d} + \frac{16(bc-ad)^{9/4}\sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{7b^{1/4}d^3\sqrt{a+bx}}$$

Result (type 5, 106 leaves):

$$\frac{1}{7d^3\sqrt{a+bx}} 4(c+dx)^{1/4} \left( d(a+bx)(-2bc+3ad+bdx) + 4(bc-ad)^2 \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

**Problem 1655: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{3/4}} dx$$

Optimal (type 4, 111 leaves, 4 steps):

$$\frac{4\sqrt{a+bx}(c+dx)^{1/4}}{3d} - \frac{8(bc-ad)^{5/4}\sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{3b^{1/4}d^2\sqrt{a+bx}}$$

Result (type 5, 77 leaves):

$$\frac{4\sqrt{a+bx}(c+dx)^{1/4} \left( 1 + \frac{2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad}\right]}{\sqrt{\frac{d(a+bx)}{-bc-ad}}} \right)}{3d}$$

**Problem 1656: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{3/4}} dx$$

Optimal (type 4, 83 leaves, 3 steps):

$$\frac{4 (b c - a d)^{1/4} \sqrt{-\frac{d (a+b x)}{b c - a d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{b^{1/4} d \sqrt{a+b x}}$$

Result (type 5, 71 leaves):

$$\frac{4 \sqrt{\frac{d (a+b x)}{-b c + a d}} (c+d x)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c+d x)}{b c - a d}\right]}{d \sqrt{a+b x}}$$

**Problem 1657: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+b x)^{3/2} (c+d x)^{3/4}} dx$$

Optimal (type 4, 111 leaves, 4 steps):

$$-\frac{2 (c+d x)^{1/4}}{(b c - a d) \sqrt{a+b x}} - \frac{2 \sqrt{-\frac{d (a+b x)}{b c - a d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{b^{1/4} (b c - a d)^{3/4} \sqrt{a+b x}}$$

Result (type 5, 81 leaves):

$$-\frac{2 (c+d x)^{1/4} \left(1 + \sqrt{\frac{d (a+b x)}{-b c + a d}} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c+d x)}{b c - a d}\right]\right)}{(b c - a d) \sqrt{a+b x}}$$

**Problem 1658: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+b x)^{5/2} (c+d x)^{3/4}} dx$$

Optimal (type 4, 149 leaves, 5 steps):

$$-\frac{2 (c+d x)^{1/4}}{3 (b c - a d) (a+b x)^{3/2}} + \frac{5 d (c+d x)^{1/4}}{3 (b c - a d)^2 \sqrt{a+b x}} + \frac{5 d \sqrt{-\frac{d (a+b x)}{b c - a d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} (c+d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{3 b^{1/4} (b c - a d)^{7/4} \sqrt{a+b x}}$$

Result (type 5, 102 leaves):

$$\left( (c+dx)^{1/4} \left( -2bc+7ad+5bdx+5d(a+bx) \sqrt{\frac{d(a+bx)}{-bc+ad}} \right. \right. \\ \left. \left. \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) / \left( 3(bc-ad)^2 (a+bx)^{3/2} \right)$$

**Problem 1659: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/4}} dx$$

Optimal (type 4, 254 leaves, 10 steps):

$$-\frac{4(a+bx)^{5/2}}{d(c+dx)^{1/4}} - \frac{16b(bc-ad)\sqrt{a+bx}(c+dx)^{3/4}}{3d^3} + \frac{40b(a+bx)^{3/2}(c+dx)^{3/4}}{9d^2} + \\ \frac{1}{3d^4\sqrt{a+bx}} 32b^{1/4}(bc-ad)^{11/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right] - \\ \frac{1}{3d^4\sqrt{a+bx}} 32b^{1/4}(bc-ad)^{11/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]$$

Result (type 5, 131 leaves):

$$\frac{1}{9d^4\sqrt{a+bx}} 4(c+dx)^{3/4} \left( d(a+bx) \left( b(-3bc+4ad) + b^2dx - \frac{9(bc-ad)^2}{c+dx} \right) + \right. \\ \left. 8b(bc-ad)^2 \sqrt{\frac{d(a+bx)}{-bc+ad}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

**Problem 1660: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/4}} dx$$

Optimal (type 4, 220 leaves, 9 steps):

$$-\frac{4(a+bx)^{3/2}}{d(c+dx)^{1/4}} + \frac{24b\sqrt{a+bx}(c+dx)^{3/4}}{5d^2} - \frac{1}{5d^3\sqrt{a+bx}}$$

$$48b^{1/4}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right] +$$

$$\frac{1}{5d^3\sqrt{a+bx}}48b^{1/4}(bc-ad)^{7/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]$$

Result (type 5, 98 leaves):

$$\frac{4\sqrt{a+bx}(c+dx)^{3/4}\left(\frac{6bc-5ad+bdx}{c+dx} + \frac{4b\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right]}{\sqrt{\frac{d(a+bx)}{-bc+ad}}}\right)}{5d^2}$$

**Problem 1661: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{5/4}} dx$$

Optimal (type 4, 190 leaves, 8 steps):

$$-\frac{4\sqrt{a+bx}}{d(c+dx)^{1/4}} + \frac{8b^{1/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{d^2\sqrt{a+bx}}$$

$$\frac{8b^{1/4}(bc-ad)^{3/4}\sqrt{-\frac{d(a+bx)}{bc-ad}}\text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{d^2\sqrt{a+bx}}$$

Result (type 5, 90 leaves):

$$\frac{\left(-12d(a+bx) + 8b\sqrt{\frac{d(a+bx)}{-bc+ad}}(c+dx)\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right]\right)}{\left(3d^2\sqrt{a+bx}(c+dx)^{1/4}\right)}$$

**Problem 1662: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{5/4}} dx$$

Optimal (type 4, 197 leaves, 8 steps):

$$\frac{4\sqrt{a+bx}}{(bc-d)(c+dx)^{1/4}} - \frac{4b^{1/4}\sqrt{-\frac{d(a+bx)}{bc-d}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-d)^{1/4}}\right], -1\right]}{d(bc-d)^{1/4}\sqrt{a+bx}} +$$

$$\frac{4b^{1/4}\sqrt{-\frac{d(a+bx)}{bc-d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-d)^{1/4}}\right], -1\right]}{d(bc-d)^{1/4}\sqrt{a+bx}}$$

Result (type 5, 100 leaves):

$$\left( \frac{12d(a+bx) - 4b\sqrt{\frac{d(a+bx)}{-bc+ad}}(c+dx) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-d}\right]}{3d(bc-d)\sqrt{a+bx}(c+dx)^{1/4}} \right) /$$

**Problem 1663: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{3/2}(c+dx)^{5/4}} dx$$

Optimal (type 4, 222 leaves, 9 steps):

$$-\frac{2}{(bc-d)\sqrt{a+bx}(c+dx)^{1/4}} - \frac{6d\sqrt{a+bx}}{(bc-d)^2(c+dx)^{1/4}} +$$

$$\frac{6b^{1/4}\sqrt{-\frac{d(a+bx)}{bc-d}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-d)^{1/4}}\right], -1\right]}{(bc-d)^{5/4}\sqrt{a+bx}} -$$

$$\frac{6b^{1/4}\sqrt{-\frac{d(a+bx)}{bc-d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-d)^{1/4}}\right], -1\right]}{(bc-d)^{5/4}\sqrt{a+bx}}$$

Result (type 5, 99 leaves):

$$\left( \frac{-4ad - 2b(c+3dx) + 2b\sqrt{\frac{d(a+bx)}{-bc+ad}}(c+dx) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-d}\right]}{(bc-d)^2\sqrt{a+bx}(c+dx)^{1/4}} \right) /$$

**Problem 1664: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{5/2}(c+dx)^{5/4}} dx$$

Optimal (type 4, 261 leaves, 10 steps):

$$\begin{aligned}
 & -\frac{2}{3 (bc - ad) (a + bx)^{3/2} (c + dx)^{1/4}} + \frac{7d}{3 (bc - ad)^2 \sqrt{a + bx} (c + dx)^{1/4}} + \\
 & \frac{7d^2 \sqrt{a + bx}}{(bc - ad)^3 (c + dx)^{1/4}} - \frac{7b^{1/4} d \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{(bc - ad)^{9/4} \sqrt{a + bx}} + \\
 & \frac{7b^{1/4} d \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{(bc - ad)^{9/4} \sqrt{a + bx}}
 \end{aligned}$$

Result (type 5, 139 leaves):

$$\left( -12a^2d^2 - abd(11c + 35dx) + b^2(2c^2 - 7cdx - 21d^2x^2) + \right. \\
 \left. 7bd(a + bx) \sqrt{\frac{d(a + bx)}{-bc + ad}} (c + dx) \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c + dx)}{bc - ad}\right] \right) / \\
 (3(-bc + ad)^3 (a + bx)^{3/2} (c + dx)^{1/4})$$

**Problem 1665: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + bx)^{7/2}}{(c + dx)^{7/4}} dx$$

Optimal (type 4, 207 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{4(a + bx)^{7/2}}{3d(c + dx)^{3/4}} + \frac{160b(bc - ad)^2 \sqrt{a + bx} (c + dx)^{1/4}}{33d^4} - \\
 & \frac{80b(bc - ad)(a + bx)^{3/2} (c + dx)^{1/4}}{33d^3} + \frac{56b(a + bx)^{5/2} (c + dx)^{1/4}}{33d^2} - \frac{1}{33d^5 \sqrt{a + bx}} \\
 & 320b^{3/4} (bc - ad)^{13/4} \sqrt{-\frac{d(a + bx)}{bc - ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c + dx)^{1/4}}{(bc - ad)^{1/4}}\right], -1\right]
 \end{aligned}$$

Result (type 5, 181 leaves):

$$\frac{1}{33 d^5 \sqrt{a+bx}}$$

$$4 (c+dx)^{1/4} \left( \frac{1}{c+dx} d (a+bx) \left( 11 (bc-ad)^3 + b (29 b^2 c^2 - 67 abc d + 41 a^2 d^2) (c+dx) - \right. \right.$$

$$\left. \left. 3 b^2 d (3bc-5ad) x (c+dx) + 3 b^3 d^2 x^2 (c+dx) \right) - \right.$$

$$\left. 80 b (bc-ad)^3 \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad} \right] \right)$$

**Problem 1666: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/4}} dx$$

Optimal (type 4, 137 leaves, 5 steps):

$$-\frac{4(a+bx)^{3/2}}{3d(c+dx)^{3/4}} + \frac{8b\sqrt{a+bx}(c+dx)^{1/4}}{3d^2} - \frac{1}{3d^3\sqrt{a+bx}}$$

$$16b^{3/4}(bc-ad)^{5/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}} \right], -1 \right]$$

Result (type 5, 98 leaves):

$$\frac{4\sqrt{a+bx}(c+dx)^{1/4} \left( \frac{2bc-ad+bdx}{c+dx} + \frac{4b \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad} \right]}{\sqrt{\frac{d(a+bx)}{-bc+ad}}} \right)}{3d^2}$$

**Problem 1667: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{7/4}} dx$$

Optimal (type 4, 111 leaves, 4 steps):

$$-\frac{4\sqrt{a+bx}}{3d(c+dx)^{3/4}} + \frac{8b^{3/4}(bc-ad)^{1/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF} \left[ \operatorname{ArcSin} \left[ \frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}} \right], -1 \right]}{3d^2\sqrt{a+bx}}$$

Result (type 5, 90 leaves):

$$\left( \frac{-4 d (a + b x) + 8 b \sqrt{\frac{d (a + b x)}{-b c + a d}} (c + d x) \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right]}{(3 d^2 \sqrt{a + b x} (c + d x)^{3/4})} \right) /$$

**Problem 1668: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{a + b x} (c + d x)^{7/4}} dx$$

Optimal (type 4, 118 leaves, 4 steps):

$$\frac{4 \sqrt{a + b x}}{3 (b c - a d) (c + d x)^{3/4}} + \frac{4 b^{3/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{3 d (b c - a d)^{3/4} \sqrt{a + b x}}$$

Result (type 5, 98 leaves):

$$\left( \frac{4 \left( d (a + b x) + b \sqrt{\frac{d (a + b x)}{-b c + a d}} (c + d x) \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)}{(3 d (b c - a d) \sqrt{a + b x} (c + d x)^{3/4})} \right) /$$

**Problem 1669: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b x)^{3/2} (c + d x)^{7/4}} dx$$

Optimal (type 4, 146 leaves, 5 steps):

$$-\frac{2}{(b c - a d) \sqrt{a + b x} (c + d x)^{3/4}} - \frac{10 d \sqrt{a + b x}}{3 (b c - a d)^2 (c + d x)^{3/4}} - \frac{10 b^{3/4} \sqrt{-\frac{d (a + b x)}{b c - a d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{3 (b c - a d)^{7/4} \sqrt{a + b x}}$$

Result (type 5, 102 leaves):

$$-\left( \left( \left( 2 \left( 3 b c + 2 a d + 5 b d x + 5 b \sqrt{\frac{d (a + b x)}{-b c + a d}} (c + d x) \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \right) / \left( 3 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{3/4} \right) \right) \right)$$



### Problem 1670: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+bx)^{5/2} (c+dx)^{7/4}} dx$$

Optimal (type 4, 178 leaves, 6 steps):

$$-\frac{2}{3(b c-a d)(a+b x)^{3/2}(c+d x)^{3/4}}+\frac{3 d}{(b c-a d)^2 \sqrt{a+b x}(c+d x)^{3/4}}+\frac{5 d^2 \sqrt{a+b x}}{(b c-a d)^3(c+d x)^{3/4}}+\frac{5 b^{3/4} d \sqrt{-\frac{d(a+b x)}{b c-a d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+d x)^{1/4}}{(b c-a d)^{1/4}}\right],-1\right]}{(b c-a d)^{11/4} \sqrt{a+b x}}$$

Result (type 5, 139 leaves):

$$\left(-4 a^2 d^2-a b d(13 c+21 d x)+b^2\left(2 c^2-9 c d x-15 d^2 x^2\right)-15 b d(a+b x) \sqrt{\frac{d(a+b x)}{-b c+a d}}(c+d x) \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, \frac{b(c+d x)}{b c-a d}\right]\right) / \left(3(-b c+a d)^3(a+b x)^{3/2}(c+d x)^{3/4}\right)$$

### Problem 1671: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx)^{7/2}}{(c+dx)^{9/4}} dx$$

Optimal (type 4, 286 leaves, 11 steps):

$$-\frac{4(a+b x)^{7/2}}{5 d(c+d x)^{5/4}}-\frac{56 b(a+b x)^{5/2}}{5 d^2(c+d x)^{1/4}}-\frac{224 b^2(b c-a d) \sqrt{a+b x}(c+d x)^{3/4}}{15 d^4}+\frac{112 b^2(a+b x)^{3/2}(c+d x)^{3/4}}{9 d^3}+\frac{1}{15 d^5 \sqrt{a+b x}}-448 b^{5/4}(b c-a d)^{11/4} \sqrt{-\frac{d(a+b x)}{b c-a d}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+d x)^{1/4}}{(b c-a d)^{1/4}}\right],-1\right]-\frac{1}{15 d^5 \sqrt{a+b x}} 448 b^{5/4}(b c-a d)^{11/4} \sqrt{-\frac{d(a+b x)}{b c-a d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+d x)^{1/4}}{(b c-a d)^{1/4}}\right],-1\right]$$

Result (type 5, 169 leaves):

$$\frac{1}{45 d^5 \sqrt{a+bx}} 4 (c+dx)^{3/4} \left( \frac{1}{(c+dx)^2} d (a+bx) \right. \\ \left. (9 (bc-ad)^3 - 153 b (bc-ad)^2 (c+dx) - b^2 (24bc-29ad) (c+dx)^2 + 5b^3 dx (c+dx)^2) + \right. \\ \left. 112 b^2 (bc-ad)^2 \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

**Problem 1672: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{9/4}} dx$$

Optimal (type 4, 248 leaves, 10 steps):

$$-\frac{4(a+bx)^{5/2}}{5d(c+dx)^{5/4}} - \frac{8b(a+bx)^{3/2}}{d^2(c+dx)^{1/4}} + \frac{48b^2\sqrt{a+bx}(c+dx)^{3/4}}{5d^3} - \frac{1}{5d^4\sqrt{a+bx}} \\ 96b^{5/4}(bc-ad)^{7/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right] + \\ \frac{1}{5d^4\sqrt{a+bx}} 96b^{5/4}(bc-ad)^{7/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]$$

Result (type 5, 141 leaves):

$$\frac{1}{5d^4\sqrt{a+bx}} 4 (c+dx)^{3/4} \left( -\frac{d(a+bx) \left( (bc-ad)^2 - 12b(bc-ad)(c+dx) - b^2(c+dx)^2 \right)}{(c+dx)^2} \right. \\ \left. 8b^2(bc-ad) \sqrt{\frac{d(a+bx)}{-bc+ad}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

**Problem 1673: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{9/4}} dx$$

Optimal (type 4, 222 leaves, 9 steps):

$$\begin{aligned}
 & -\frac{4(a+bx)^{3/2}}{5d(c+dx)^{5/4}} - \frac{24b\sqrt{a+bx}}{5d^2(c+dx)^{1/4}} + \frac{1}{5d^3\sqrt{a+bx}} \\
 & 48b^{5/4}(bc-ad)^{3/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right] - \\
 & \frac{1}{5d^3\sqrt{a+bx}} 48b^{5/4}(bc-ad)^{3/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]
 \end{aligned}$$

Result (type 5, 107 leaves):

$$\left( -4d(a+bx)(6bc+ad+7bdx) + 16b^2 \sqrt{\frac{d(a+bx)}{-bc+ad}} (c+dx)^2 \right. \\
 \left. \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right) / (5d^3\sqrt{a+bx}(c+dx)^{5/4})$$

**Problem 1674: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{9/4}} dx$$

Optimal (type 4, 232 leaves, 9 steps):

$$\begin{aligned}
 & -\frac{4\sqrt{a+bx}}{5d(c+dx)^{5/4}} + \frac{8b\sqrt{a+bx}}{5d(bc-ad)(c+dx)^{1/4}} - \\
 & \frac{8b^{5/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{5d^2(bc-ad)^{1/4}\sqrt{a+bx}} + \\
 & \frac{8b^{5/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4}(c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{5d^2(bc-ad)^{1/4}\sqrt{a+bx}}
 \end{aligned}$$

Result (type 5, 116 leaves):

$$\left( -12d(a+bx)(ad+b(c+2dx)) + \right. \\
 \left. 8b^2 \sqrt{\frac{d(a+bx)}{-bc+ad}} (c+dx)^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right) / \\
 (15d^2(-bc+ad)\sqrt{a+bx}(c+dx)^{5/4})$$

**Problem 1675: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{a+bx} (c+dx)^{9/4}} dx$$

Optimal (type 4, 236 leaves, 9 steps):

$$\frac{4 \sqrt{a+bx}}{5 (bc-ad) (c+dx)^{5/4}} + \frac{12 b \sqrt{a+bx}}{5 (bc-ad)^2 (c+dx)^{1/4}} - \frac{12 b^{5/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4} (c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{5 d (bc-ad)^{5/4} \sqrt{a+bx}} + \frac{12 b^{5/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{5 d (bc-ad)^{5/4} \sqrt{a+bx}}$$

Result (type 5, 115 leaves):

$$-\left( \left( 4 \left( d (a+bx) (-4bc+ad-3bdx) + b^2 \sqrt{\frac{d(a+bx)}{-bc+ad}} (c+dx)^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) / \left( 5 d (bc-ad)^2 \sqrt{a+bx} (c+dx)^{5/4} \right) \right)$$

**Problem 1676: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{3/2} (c+dx)^{9/4}} dx$$

Optimal (type 4, 262 leaves, 10 steps):

$$-\frac{2}{(bc-ad) \sqrt{a+bx} (c+dx)^{5/4}} - \frac{14 d \sqrt{a+bx}}{5 (bc-ad)^2 (c+dx)^{5/4}} - \frac{42 b d \sqrt{a+bx}}{5 (bc-ad)^3 (c+dx)^{1/4}} + \frac{42 b^{5/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4} (c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{5 (bc-ad)^{9/4} \sqrt{a+bx}} - \frac{42 b^{5/4} \sqrt{-\frac{d(a+bx)}{bc-ad}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c+dx)^{1/4}}{(bc-ad)^{1/4}}\right], -1\right]}{5 (bc-ad)^{9/4} \sqrt{a+bx}}$$

Result (type 5, 138 leaves):

$$\left( -4 a^2 d^2 + 4 a b d (9 c + 7 d x) + 2 b^2 (5 c^2 + 28 c d x + 21 d^2 x^2) - \right. \\ \left. 14 b^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} (c + d x)^2 \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) / \\ (5 (-b c + a d)^3 \sqrt{a + b x} (c + d x)^{5/4})$$

**Problem 1677: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b x)^{5/2} (c + d x)^{9/4}} dx$$

Optimal (type 4, 303 leaves, 11 steps):

$$-\frac{2}{3 (b c - a d) (a + b x)^{3/2} (c + d x)^{5/4}} + \frac{11 d}{3 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{5/4}} + \\ \frac{77 d^2 \sqrt{a + b x}}{15 (b c - a d)^3 (c + d x)^{5/4}} + \frac{77 b d^2 \sqrt{a + b x}}{5 (b c - a d)^4 (c + d x)^{1/4}} - \\ \frac{77 b^{5/4} d \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticE}\left[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{5 (b c - a d)^{13/4} \sqrt{a + b x}} + \\ \frac{77 b^{5/4} d \sqrt{-\frac{d (a + b x)}{b c - a d}} \text{EllipticF}\left[\text{ArcSin}\left[\frac{b^{1/4} (c + d x)^{1/4}}{(b c - a d)^{1/4}}\right], -1\right]}{5 (b c - a d)^{13/4} \sqrt{a + b x}}$$

Result (type 5, 156 leaves):

$$\left( (c + d x)^{3/4} \left( 75 b^2 d - \frac{10 b^2 (b c - a d)}{a + b x} + \frac{12 d^2 (b c - a d) (a + b x)}{(c + d x)^2} + \frac{156 b d^2 (a + b x)}{c + d x} - \right. \right. \\ \left. \left. 77 b^2 d \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \right) / (15 (b c - a d)^4 \sqrt{a + b x})$$

**Problem 1678: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^{3/4} (c + d x)^{5/4} dx$$

Optimal (type 3, 205 leaves, 8 steps):

$$\frac{5 (b c - a d)^2 (a + b x)^{3/4} (c + d x)^{1/4}}{96 b^2 d} + \frac{5 (b c - a d) (a + b x)^{7/4} (c + d x)^{1/4}}{24 b^2} + \frac{(a + b x)^{7/4} (c + d x)^{5/4}}{3 b} + \frac{5 (b c - a d)^3 \operatorname{ArcTan}\left[\frac{d^{1/4} (a + b x)^{1/4}}{b^{1/4} (c + d x)^{1/4}}\right]}{64 b^{9/4} d^{7/4}} - \frac{5 (b c - a d)^3 \operatorname{ArcTanh}\left[\frac{d^{1/4} (a + b x)^{1/4}}{b^{1/4} (c + d x)^{1/4}}\right]}{64 b^{9/4} d^{7/4}}$$

Result (type 5, 143 leaves):

$$\left( (c + d x)^{1/4} \left( -d (a + b x) (15 a^2 d^2 - 6 a b d (7 c + 2 d x) - b^2 (5 c^2 + 52 c d x + 32 d^2 x^2)) - 15 (b c - a d)^3 \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right) \right) / (96 b^2 d^2 (a + b x)^{1/4})$$

**Problem 1679: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + d x)^{5/4}}{(a + b x)^{1/4}} dx$$

Optimal (type 3, 167 leaves, 7 steps):

$$\frac{5 (b c - a d) (a + b x)^{3/4} (c + d x)^{1/4}}{8 b^2} + \frac{(a + b x)^{3/4} (c + d x)^{5/4}}{2 b} - \frac{5 (b c - a d)^2 \operatorname{ArcTan}\left[\frac{d^{1/4} (a + b x)^{1/4}}{b^{1/4} (c + d x)^{1/4}}\right]}{16 b^{9/4} d^{3/4}} + \frac{5 (b c - a d)^2 \operatorname{ArcTanh}\left[\frac{d^{1/4} (a + b x)^{1/4}}{b^{1/4} (c + d x)^{1/4}}\right]}{16 b^{9/4} d^{3/4}}$$

Result (type 5, 111 leaves):

$$\frac{1}{8 b^2 d (a + b x)^{1/4}} (c + d x)^{1/4} \left( -d (a + b x) (-9 b c + 5 a d - 4 b d x) + 5 (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

**Problem 1680: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + d x)^{5/4}}{(a + b x)^{5/4}} dx$$

Optimal (type 3, 152 leaves, 7 steps):

$$\frac{5 d (a + b x)^{3/4} (c + d x)^{1/4}}{b^2} - \frac{4 (c + d x)^{5/4}}{b (a + b x)^{1/4}} - \frac{5 d^{1/4} (b c - a d) \operatorname{ArcTan}\left[\frac{d^{1/4} (a + b x)^{1/4}}{b^{1/4} (c + d x)^{1/4}}\right]}{2 b^{9/4}} + \frac{5 d^{1/4} (b c - a d) \operatorname{ArcTanh}\left[\frac{d^{1/4} (a + b x)^{1/4}}{b^{1/4} (c + d x)^{1/4}}\right]}{2 b^{9/4}}$$

Result (type 5, 93 leaves):

$$\frac{1}{b^2 (a+bx)^{1/4}} (c+dx)^{1/4} \left( -4bc + 5ad + bdx + 5(bc - ad) \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad} \right] \right)$$

**Problem 1681: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{9/4}} dx$$

Optimal (type 3, 134 leaves, 7 steps):

$$-\frac{4d(c+dx)^{1/4}}{b^2(a+bx)^{1/4}} - \frac{4(c+dx)^{5/4}}{5b(a+bx)^{5/4}} - \frac{2d^{5/4} \text{ArcTan} \left[ \frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}} \right]}{b^{9/4}} + \frac{2d^{5/4} \text{ArcTanh} \left[ \frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}} \right]}{b^{9/4}}$$

Result (type 5, 94 leaves):

$$-\frac{1}{5b^2(a+bx)^{5/4}} (c+dx)^{1/4} \left( 5ad + b(c+6dx) - 5d(a+bx) \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad} \right] \right)$$

**Problem 1686: Result unnecessarily involves higher level functions.**

$$\int (a+bx)^{5/4} (c+dx)^{5/4} dx$$

Optimal (type 4, 408 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{5 (bc - ad)^3 (a + bx)^{1/4} (c + dx)^{1/4}}{84 b^2 d^2} + \frac{(bc - ad)^2 (a + bx)^{5/4} (c + dx)^{1/4}}{42 b^2 d} + \\
 & \frac{(bc - ad) (a + bx)^{9/4} (c + dx)^{1/4}}{7 b^2} + \frac{2 (a + bx)^{9/4} (c + dx)^{5/4}}{7 b} + \\
 & \left( 5 (bc - ad)^{9/2} ((a + bx) (c + dx))^{3/4} \sqrt{(bc + ad + 2 b d x)^2} \right. \\
 & \left. \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right) \sqrt{\frac{(ad + b(c + 2 dx))^2}{(bc - ad)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + bx) (c + dx))^{1/4}}{\sqrt{bc - ad}} \right], \frac{1}{2} \right] \right) / \\
 & (168 \sqrt{2} b^{9/4} d^{9/4} (a + bx)^{3/4} (c + dx)^{3/4} (bc + ad + 2 b d x) \sqrt{(ad + b(c + 2 dx))^2})
 \end{aligned}$$

Result (type 5, 183 leaves):

$$\begin{aligned}
 & \left( (c + dx)^{1/4} \left( -d (a + bx) (5 a^3 d^3 - a^2 b d^2 (17 c + 2 dx) - a b^2 d (17 c^2 + 68 c dx + 36 d^2 x^2) + \right. \right. \\
 & \quad \left. \left. b^3 (5 c^3 - 2 c^2 dx - 36 c d^2 x^2 - 24 d^3 x^3) \right) + 5 (bc - ad)^4 \left( \frac{d (a + bx)}{-bc + ad} \right)^{3/4} \right. \\
 & \left. \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b (c + dx)}{bc - ad} \right] \right) / (84 b^2 d^3 (a + bx)^{3/4})
 \end{aligned}$$

**Problem 1687: Result unnecessarily involves higher level functions.**

$$\int (a + bx)^{1/4} (c + dx)^{5/4} dx$$

Optimal (type 4, 370 leaves, 6 steps):



$$\begin{aligned}
 & \frac{(bc-ad)^2 (a+bx)^{1/4} (c+dx)^{1/4}}{6b^2d} + \frac{(bc-ad) (a+bx)^{5/4} (c+dx)^{1/4}}{3b^2} + \\
 & \frac{2(a+bx)^{5/4} (c+dx)^{5/4}}{5b} - \left( (bc-ad)^{7/2} ((a+bx)(c+dx))^{3/4} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & (12\sqrt{2} b^{9/4} d^{5/4} (a+bx)^{3/4} (c+dx)^{3/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2})
 \end{aligned}$$

Result (type 5, 142 leaves):

$$\begin{aligned}
 & \left( (c+dx)^{1/4} \left( -d(a+bx) (5a^2d^2 - 2abd(6c+dx) - b^2(5c^2 + 22cdx + 12d^2x^2)) - 5(bc-ad)^3 \right. \right. \\
 & \left. \left. \left( \frac{d(a+bx)}{-bc+ad} \right)^{3/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad} \right] \right) \right) / (30b^2d^2(a+bx)^{3/4})
 \end{aligned}$$

**Problem 1688: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{5/4}}{(a+bx)^{3/4}} dx$$

Optimal (type 4, 332 leaves, 5 steps):

$$\frac{5 (bc - ad) (a + bx)^{1/4} (c + dx)^{1/4}}{3 b^2} + \frac{2 (a + bx)^{1/4} (c + dx)^{5/4}}{3 b} +$$

$$\left( 5 (bc - ad)^{5/2} ((a + bx) (c + dx))^{3/4} \sqrt{(bc + ad + 2 b d x)^2} \right.$$

$$\left. \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right) \sqrt{\frac{(ad + b (c + 2 dx))^2}{(bc - ad)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right)^2}} \right.$$

$$\left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + bx) (c + dx))^{1/4}}{\sqrt{bc - ad}} \right], \frac{1}{2} \right] \right/$$

$$\left( 6 \sqrt{2} b^{9/4} d^{1/4} (a + bx)^{3/4} (c + dx)^{3/4} (bc + ad + 2 b d x) \sqrt{(ad + b (c + 2 dx))^2} \right)$$

Result (type 5, 111 leaves):

$$\frac{1}{3 b^2 d (a + b x)^{3/4}} (c + d x)^{1/4} \left( -d (a + b x) (-7 b c + 5 a d - 2 b d x) + \right.$$

$$\left. 5 (bc - ad)^2 \left( \frac{d (a + b x)}{-bc + ad} \right)^{3/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b (c + d x)}{bc - ad} \right] \right)$$

**Problem 1689: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + d x)^{5/4}}{(a + b x)^{7/4}} dx$$

Optimal (type 4, 325 leaves, 5 steps):

$$\frac{10 d (a+b x)^{1/4} (c+d x)^{1/4}}{3 b^2} - \frac{4 (c+d x)^{5/4}}{3 b (a+b x)^{3/4}} +$$

$$\left( 5 d^{3/4} (b c-a d)^{3/2} ((a+b x)(c+d x))^{3/4} \sqrt{(b c+a d+2 b d x)^2} \right.$$

$$\left. \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x)(c+d x)}}{b c-a d} \right) \sqrt{\frac{(a d+b(c+2 d x))^2}{(b c-a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x)(c+d x)}}{b c-a d} \right)^2}} \right.$$

$$\left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+b x)(c+d x))^{1/4}}{\sqrt{b c-a d}} \right], \frac{1}{2} \right] \right) /$$

$$\left( 3 \sqrt{2} b^{9/4} (a+b x)^{3/4} (c+d x)^{3/4} (b c+a d+2 b d x) \sqrt{(a d+b(c+2 d x))^2} \right)$$

Result (type 5, 95 leaves):

$$-\frac{1}{3 b^2 (a+b x)^{3/4}}$$

$$+ 2 (c+d x)^{1/4} \left( 2 b c-5 a d-3 b d x + \frac{5 d (a+b x) \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b(c+d x)}{b c-a d} \right]}{\left( \frac{d(a+b x)}{-b c+a d} \right)^{1/4}} \right)$$

**Problem 1690: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+d x)^{5/4}}{(a+b x)^{11/4}} dx$$

Optimal (type 4, 325 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{20 d (c+d x)^{1/4}}{21 b^2 (a+b x)^{3/4}} - \frac{4 (c+d x)^{5/4}}{7 b (a+b x)^{7/4}} + \\
 & \left( 5 \sqrt{2} d^{7/4} \sqrt{b c-a d} ((a+b x)(c+d x))^{3/4} \sqrt{(b c+a d+2 b d x)^2} \right. \\
 & \left. \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x)(c+d x)}}{b c-a d} \right) \sqrt{\frac{(a d+b(c+2 d x))^2}{(b c-a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x)(c+d x)}}{b c-a d} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+b x)(c+d x))^{1/4}}{\sqrt{b c-a d}} \right], \frac{1}{2} \right] \right) / \\
 & \left( 21 b^{9/4} (a+b x)^{3/4} (c+d x)^{3/4} (b c+a d+2 b d x) \sqrt{(a d+b(c+2 d x))^2} \right)
 \end{aligned}$$

Result (type 5, 95 leaves):

$$\begin{aligned}
 & -\frac{1}{21 b^2 (a+b x)^{7/4}} 4 (c+d x)^{1/4} \\
 & \left( 3 b c+5 a d+8 b d x-5 d (a+b x) \left( \frac{d(a+b x)}{-b c+a d} \right)^{3/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b(c+d x)}{b c-a d} \right] \right)
 \end{aligned}$$

**Problem 1691: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+d x)^{5/4}}{(a+b x)^{15/4}} dx$$

Optimal (type 4, 363 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{20 d (c+d x)^{1/4}}{77 b^2 (a+b x)^{7/4}} - \frac{20 d^2 (c+d x)^{1/4}}{231 b^2 (b c-a d) (a+b x)^{3/4}} - \\
 & \frac{4 (c+d x)^{5/4}}{11 b (a+b x)^{11/4}} - \left( 10 \sqrt{2} d^{11/4} ((a+b x) (c+d x))^{3/4} \sqrt{(b c+a d+2 b d x)^2} \right. \\
 & \left. \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c-a d} \right) \sqrt{\frac{(a d+b (c+2 d x))^2}{(b c-a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+b x) (c+d x)}}{b c-a d} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+b x) (c+d x))^{1/4}}{\sqrt{b c-a d}} \right], \frac{1}{2} \right] \right) / \\
 & \left( 231 b^{9/4} \sqrt{b c-a d} (a+b x)^{3/4} (c+d x)^{3/4} (b c+a d+2 b d x) \sqrt{(a d+b (c+2 d x))^2} \right)
 \end{aligned}$$

Result (type 5, 140 leaves):

$$\begin{aligned}
 & \left( 4 (c+d x)^{1/4} \right. \\
 & \left. \left( -10 a^2 d^2 - 2 a b d (3 c+13 d x) + b^2 (21 c^2+36 c d x+5 d^2 x^2) + 10 d^2 (a+b x)^2 \left( \frac{d (a+b x)}{-b c+a d} \right)^{3/4} \right. \right. \\
 & \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b (c+d x)}{b c-a d} \right] \right) \right) / \left( 231 b^2 (-b c+a d) (a+b x)^{11/4} \right)
 \end{aligned}$$

**Problem 1692: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+d x)^{5/4}}{(a+b x)^{19/4}} dx$$

Optimal (type 4, 401 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{4 d (c + d x)^{1/4}}{33 b^2 (a + b x)^{11/4}} - \frac{4 d^2 (c + d x)^{1/4}}{231 b^2 (b c - a d) (a + b x)^{7/4}} + \frac{8 d^3 (c + d x)^{1/4}}{231 b^2 (b c - a d)^2 (a + b x)^{3/4}} - \\
 & \frac{4 (c + d x)^{5/4}}{15 b (a + b x)^{15/4}} + \left( 4 \sqrt{2} d^{15/4} ((a + b x) (c + d x))^{3/4} \sqrt{(b c + a d + 2 b d x)^2} \right. \\
 & \left. \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right) \sqrt{\frac{(a d + b (c + 2 d x))^2}{(b c - a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}} \right], \frac{1}{2} \right] \right) / \\
 & \left( 231 b^{9/4} (b c - a d)^{3/2} (a + b x)^{3/4} (c + d x)^{3/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right)
 \end{aligned}$$

Result (type 5, 179 leaves):

$$\begin{aligned}
 & \left( 4 (c + d x)^{1/4} \left( -20 a^3 d^3 - 12 a^2 b d^2 (c + 6 d x) + a b^2 d (119 c^2 + 214 c d x + 35 d^2 x^2) - \right. \right. \\
 & \left. \left. b^3 (77 c^3 + 112 c^2 d x + 5 c d^2 x^2 - 10 d^3 x^3) + 20 d^3 (a + b x)^3 \left( \frac{d (a + b x)}{-b c + a d} \right)^{3/4} \right. \right. \\
 & \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b (c + d x)}{b c - a d} \right] \right) \right) / \left( 1155 b^2 (b c - a d)^2 (a + b x)^{15/4} \right)
 \end{aligned}$$

**Problem 1693: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{5/4}}{(c + d x)^{1/4}} dx$$

Optimal (type 3, 167 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{5 (b c - a d) (a + b x)^{1/4} (c + d x)^{3/4}}{8 d^2} + \frac{(a + b x)^{5/4} (c + d x)^{3/4}}{2 d} + \\
 & \frac{5 (b c - a d)^2 \text{ArcTan} \left[ \frac{d^{1/4} (a + b x)^{1/4}}{b^{1/4} (c + d x)^{1/4}} \right]}{16 b^{3/4} d^{9/4}} + \frac{5 (b c - a d)^2 \text{ArcTanh} \left[ \frac{d^{1/4} (a + b x)^{1/4}}{b^{1/4} (c + d x)^{1/4}} \right]}{16 b^{3/4} d^{9/4}}
 \end{aligned}$$

Result (type 5, 108 leaves):

$$\frac{1}{24 d^3 (a+bx)^{3/4} (c+dx)^{3/4}} \left( 3 d (a+bx) (-5bc+9ad+4bdx) + 5 (bc-ad)^2 \left( \frac{d(a+bx)}{-bc+ad} \right)^{3/4} \text{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad} \right] \right)$$

Problem 1694: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx)^{1/4}}{(c+dx)^{1/4}} dx$$

Optimal (type 3, 127 leaves, 6 steps):

$$\frac{(a+bx)^{1/4} (c+dx)^{3/4}}{d} - \frac{(bc-ad) \text{ArcTan} \left[ \frac{d^{1/4} (a+bx)^{1/4}}{b^{1/4} (c+dx)^{1/4}} \right]}{2 b^{3/4} d^{5/4}} - \frac{(bc-ad) \text{ArcTanh} \left[ \frac{d^{1/4} (a+bx)^{1/4}}{b^{1/4} (c+dx)^{1/4}} \right]}{2 b^{3/4} d^{5/4}}$$

Result (type 5, 76 leaves):

$$\frac{(a+bx)^{1/4} (c+dx)^{3/4} \left( 3 + \frac{\text{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad} \right]}{\left( \frac{d(a+bx)}{-bc+ad} \right)^{1/4}} \right)}{3 d}$$

Problem 1695: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+bx)^{3/4} (c+dx)^{1/4}} dx$$

Optimal (type 3, 85 leaves, 5 steps):

$$\frac{2 \text{ArcTan} \left[ \frac{d^{1/4} (a+bx)^{1/4}}{b^{1/4} (c+dx)^{1/4}} \right]}{b^{3/4} d^{1/4}} + \frac{2 \text{ArcTanh} \left[ \frac{d^{1/4} (a+bx)^{1/4}}{b^{1/4} (c+dx)^{1/4}} \right]}{b^{3/4} d^{1/4}}$$

Result (type 5, 73 leaves):

$$\frac{4 \left( \frac{d(a+bx)}{-bc+ad} \right)^{3/4} (c+dx)^{3/4} \text{Hypergeometric2F1} \left[ \frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad} \right]}{3 d (a+bx)^{3/4}}$$

Problem 1700: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx)^{7/4}}{(c+dx)^{1/4}} dx$$

Optimal (type 4, 751 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{7 (bc - ad) (a + bx)^{3/4} (c + dx)^{3/4}}{15 d^2} + \frac{2 (a + bx)^{7/4} (c + dx)^{3/4}}{5 d} + \\
 & \left( 7 (bc - ad) \sqrt{(a + bx) (c + dx)} \sqrt{(bc + ad + 2 b dx)^2} \sqrt{(ad + b (c + 2 dx))^2} \right) / \\
 & \left( 10 \sqrt{b} d^{5/2} (a + bx)^{1/4} (c + dx)^{1/4} (bc + ad + 2 b dx) \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right) \right) - \\
 & \left( 7 (bc - ad)^{7/2} ((a + bx) (c + dx))^{1/4} \sqrt{(bc + ad + 2 b dx)^2} \right. \\
 & \left. \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right) \sqrt{\frac{(ad + b (c + 2 dx))^2}{(bc - ad)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + bx) (c + dx))^{1/4}}{\sqrt{bc - ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( 10 \sqrt{2} b^{3/4} d^{11/4} (a + bx)^{1/4} (c + dx)^{1/4} (bc + ad + 2 b dx) \sqrt{(ad + b (c + 2 dx))^2} \right) + \\
 & \left( 7 (bc - ad)^{7/2} ((a + bx) (c + dx))^{1/4} \sqrt{(bc + ad + 2 b dx)^2} \right. \\
 & \left. \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right) \sqrt{\frac{(ad + b (c + 2 dx))^2}{(bc - ad)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + bx) (c + dx))^{1/4}}{\sqrt{bc - ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( 20 \sqrt{2} b^{3/4} d^{11/4} (a + bx)^{1/4} (c + dx)^{1/4} (bc + ad + 2 b dx) \sqrt{(ad + b (c + 2 dx))^2} \right)
 \end{aligned}$$

Result (type 5, 107 leaves):



$$\frac{1}{15 d^3 (a + b x)^{1/4}} (c + d x)^{3/4} \left( d (a + b x) (-7 b c + 13 a d + 6 b d x) + \right. \\ \left. 7 (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

Problem 1701: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{3/4}}{(c + d x)^{1/4}} dx$$

Optimal (type 4, 705 leaves, 6 steps):

$$\begin{aligned}
 & \frac{2 (a+bx)^{3/4} (c+dx)^{3/4}}{3d} - \left( \sqrt{(a+bx)(c+dx)} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2} \right) / \\
 & \left( \sqrt{b} d^{3/2} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \right) + \\
 & \left( (bc-ad)^{5/2} ((a+bx)(c+dx))^{1/4} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( \sqrt{2} b^{3/4} d^{7/4} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right) - \\
 & \left( (bc-ad)^{5/2} ((a+bx)(c+dx))^{1/4} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( 2\sqrt{2} b^{3/4} d^{7/4} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 76 leaves):

$$\frac{2 (a+bx)^{3/4} (c+dx)^{3/4} \left( 1 + \frac{\text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad} \right]}{\left( \frac{d(a-bx)}{-bc+ad} \right)^{3/4}} \right)}{3d}$$

Problem 1702: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{1/4} (c + d x)^{1/4}} dx$$

Optimal (type 4, 688 leaves, 5 steps):

$$\begin{aligned}
 & \left( 2 \sqrt{(a+bx)(c+dx)} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2} \right) / \left( \sqrt{b} \sqrt{d} (bc-ad) \right. \\
 & \quad \left. (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \right) - \\
 & \left( \sqrt{2} (bc-ad)^{3/2} ((a+bx)(c+dx))^{1/4} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \quad \left. \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \quad \left. \text{EllipticE} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( b^{3/4} d^{3/4} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right) + \\
 & \left( (bc-ad)^{3/2} ((a+bx)(c+dx))^{1/4} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \quad \left. \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \quad \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( \sqrt{2} b^{3/4} d^{3/4} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 73 leaves):

$$\frac{4 \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/4} (c+dx)^{3/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad} \right]}{3d(a+bx)^{1/4}}$$

Problem 1703: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{5/4} (c + d x)^{1/4}} dx$$

Optimal (type 4, 718 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{4 (c+dx)^{3/4}}{(bc-ad)(a+bx)^{1/4}} + \\
 & \left( 4\sqrt{d} \sqrt{(a+bx)(c+dx)} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2} \right) / \\
 & \left( \sqrt{b} (bc-ad)^2 (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \right) - \\
 & \left( 2\sqrt{2} d^{1/4} \sqrt{bc-ad} ((a+bx)(c+dx))^{1/4} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( b^{3/4} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right) + \\
 & \left( \sqrt{2} d^{1/4} \sqrt{bc-ad} ((a+bx)(c+dx))^{1/4} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( b^{3/4} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 84 leaves):

$$\left( 4 (c + dx)^{3/4} \left( -3 + 2 \left( \frac{d (a + bx)}{-bc + ad} \right)^{1/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + dx)}{bc - ad} \right] \right) \right) / \left( 3 (bc - ad) (a + bx)^{1/4} \right)$$

**Problem 1704: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + bx)^{9/4} (c + dx)^{1/4}} dx$$

Optimal (type 4, 760 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{4(c+dx)^{3/4}}{5(bc-ad)(a+bx)^{5/4}} + \frac{8d(c+dx)^{3/4}}{5(bc-ad)^2(a+bx)^{1/4}} - \\
 & \left( 8d^{3/2} \sqrt{(a+bx)(c+dx)} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2} \right) / \left( 5\sqrt{b}(bc-ad)^3 \right. \\
 & \quad \left. (a+bx)^{1/4}(c+dx)^{1/4}(bc+ad+2bdx) \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \right) + \\
 & \left( 4\sqrt{2}d^{5/4}((a+bx)(c+dx))^{1/4} \sqrt{(bc+ad+2bdx)^2} \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \right. \\
 & \quad \left. \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \quad \left. \text{EllipticE} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2}b^{1/4}d^{1/4}((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( 5b^{3/4}\sqrt{bc-ad}(a+bx)^{1/4}(c+dx)^{1/4}(bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right) - \\
 & \left( 2\sqrt{2}d^{5/4}((a+bx)(c+dx))^{1/4} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \quad \left. \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \quad \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2}b^{1/4}d^{1/4}((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( 5b^{3/4}\sqrt{bc-ad}(a+bx)^{1/4}(c+dx)^{1/4}(bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result(type 5, 102 leaves):



$$- \left( \left( 4 (c+dx)^{3/4} \left( -9ad + 3b(c-2dx) + 4d(a+bx) \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/4} \right. \right. \right. \\ \left. \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad} \right] \right) \right) \right) / \left( 15 (bc-ad)^2 (a+bx)^{5/4} \right)$$

**Problem 1705: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{7/4}}{(c+dx)^{3/4}} dx$$

Optimal (type 3, 167 leaves, 7 steps):

$$- \frac{7 (bc-ad) (a+bx)^{3/4} (c+dx)^{1/4}}{8d^2} + \frac{(a+bx)^{7/4} (c+dx)^{1/4}}{2d} - \\ \frac{21 (bc-ad)^2 \text{ArcTan} \left[ \frac{d^{1/4} (a+bx)^{1/4}}{b^{1/4} (c+dx)^{1/4}} \right]}{16 b^{1/4} d^{11/4}} + \frac{21 (bc-ad)^2 \text{ArcTanh} \left[ \frac{d^{1/4} (a+bx)^{1/4}}{b^{1/4} (c+dx)^{1/4}} \right]}{16 b^{1/4} d^{11/4}}$$

Result (type 5, 107 leaves):

$$\frac{1}{8d^3 (a+bx)^{1/4}} (c+dx)^{1/4} \left( d(a+bx) (-7bc + 11ad + 4bdx) + \right. \\ \left. 21 (bc-ad)^2 \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad} \right] \right)$$

**Problem 1706: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{3/4}}{(c+dx)^{3/4}} dx$$

Optimal (type 3, 127 leaves, 6 steps):

$$\frac{(a+bx)^{3/4} (c+dx)^{1/4}}{d} + \frac{3 (bc-ad) \text{ArcTan} \left[ \frac{d^{1/4} (a+bx)^{1/4}}{b^{1/4} (c+dx)^{1/4}} \right]}{2 b^{1/4} d^{7/4}} - \frac{3 (bc-ad) \text{ArcTanh} \left[ \frac{d^{1/4} (a+bx)^{1/4}}{b^{1/4} (c+dx)^{1/4}} \right]}{2 b^{1/4} d^{7/4}}$$

Result (type 5, 74 leaves):

$$\frac{(a+bx)^{3/4} (c+dx)^{1/4} \left( 1 + \frac{3 \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad} \right]}{\left( \frac{d(a+bx)}{-bc+ad} \right)^{3/4}} \right)}{d}$$

**Problem 1707: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{1/4} (c+dx)^{3/4}} dx$$

Optimal (type 3, 85 leaves, 5 steps):

$$-\frac{2 \operatorname{ArcTan}\left[\frac{d^{1/4} (a+bx)^{1/4}}{b^{1/4} (c+dx)^{1/4}}\right]}{b^{1/4} d^{3/4}} + \frac{2 \operatorname{ArcTanh}\left[\frac{d^{1/4} (a+bx)^{1/4}}{b^{1/4} (c+dx)^{1/4}}\right]}{b^{1/4} d^{3/4}}$$

Result (type 5, 71 leaves):

$$\frac{4 \left(\frac{d(a+bx)}{-bc+ad}\right)^{1/4} (c+dx)^{1/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad}\right]}{d(a+bx)^{1/4}}$$

**Problem 1712: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{5/4}}{(c+dx)^{3/4}} dx$$

Optimal (type 4, 332 leaves, 5 steps):

$$-\frac{5(bc-ad)(a+bx)^{1/4}(c+dx)^{1/4}}{3d^2} + \frac{2(a+bx)^{5/4}(c+dx)^{1/4}}{3d} + \left( \frac{5(bc-ad)^{5/2}((a+bx)(c+dx))^{3/4} \sqrt{(bc+ad+2bdx)^2}}{\left(1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left(1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)^2}}}\right) \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}}\right], \frac{1}{2}\right] \sqrt{\frac{6\sqrt{2} b^{1/4} d^{9/4} (a+bx)^{3/4} (c+dx)^{3/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2}}{}}$$

Result (type 5, 107 leaves):

$$\frac{1}{3d^3(a+bx)^{3/4}}(c+dx)^{1/4} \left( d(a+bx)(-5bc+7ad+2bdx) + 5(bc-ad)^2 \left(\frac{d(a+bx)}{-bc+ad}\right)^{3/4} \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

**Problem 1713: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/4}}{(c+dx)^{3/4}} dx$$

Optimal (type 4, 295 leaves, 4 steps):

$$\frac{2 (a + b x)^{1/4} (c + d x)^{1/4}}{d} - \left( \frac{(b c - a d)^{3/2} ((a + b x) (c + d x))^{3/4} \sqrt{(b c + a d + 2 b d x)^2} \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right)}{\sqrt{(b c - a d)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + b x) (c + d x)}}{b c - a d} \right)^2}} \right. \\ \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + b x) (c + d x))^{1/4}}{\sqrt{b c - a d}} \right], \frac{1}{2} \right] \right) / \\ \left( \sqrt{2} b^{1/4} d^{5/4} (a + b x)^{3/4} (c + d x)^{3/4} (b c + a d + 2 b d x) \sqrt{(a d + b (c + 2 d x))^2} \right)$$

Result (type 5, 74 leaves):

$$\frac{2 (a + b x)^{1/4} (c + d x)^{1/4} \left( 1 + \frac{\text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b(c+dx)}{b c - a d} \right]}{\left( \frac{d(a+bx)}{-b c + a d} \right)^{1/4}} \right)}{d}$$

**Problem 1714: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b x)^{3/4} (c + d x)^{3/4}} dx$$

Optimal (type 4, 270 leaves, 3 steps):

$$\left( \sqrt{2} \sqrt{bc-ad} ((a+bx)(c+dx))^{3/4} \sqrt{(bc+ad+2bdx)^2} \right. \\ \left. \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\ \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\ \left( b^{1/4} d^{1/4} (a+bx)^{3/4} (c+dx)^{3/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right)$$

Result (type 5, 71 leaves):

$$\frac{4 \left( \frac{d(a+bx)}{-bc+ad} \right)^{3/4} (c+dx)^{1/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad} \right]}{d (a+bx)^{3/4}}$$

Problem 1715: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+bx)^{7/4} (c+dx)^{3/4}} dx$$

Optimal (type 4, 306 leaves, 4 steps):

$$\begin{aligned}
 & - \frac{4 (c + dx)^{1/4}}{3 (bc - ad) (a + bx)^{3/4}} - \\
 & \left( 2 \sqrt{2} d^{3/4} ((a + bx) (c + dx))^{3/4} \sqrt{(bc + ad + 2bdx)^2} \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right) \right. \\
 & \sqrt{\frac{(ad + b(c + 2dx))^2}{(bc - ad)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right)^2}} \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + bx) (c + dx))^{1/4}}{\sqrt{bc - ad}} \right], \frac{1}{2} \right] \right) / \\
 & (3 b^{1/4} \sqrt{bc - ad} (a + bx)^{3/4} (c + dx)^{3/4} (bc + ad + 2bdx) \sqrt{(ad + b(c + 2dx))^2})
 \end{aligned}$$

Result (type 5, 84 leaves):

$$\begin{aligned}
 & - \left( \left( 4 (c + dx)^{1/4} \left( 1 + 2 \left( \frac{d(a + bx)}{-bc + ad} \right)^{3/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b(c + dx)}{bc - ad} \right] \right) \right) / \\
 & (3 (bc - ad) (a + bx)^{3/4})
 \end{aligned}$$

**Problem 1716: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + bx)^{11/4} (c + dx)^{3/4}} dx$$

Optimal (type 4, 339 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{4(c+dx)^{1/4}}{7(bc-ad)(a+bx)^{7/4}} + \frac{8d(c+dx)^{1/4}}{7(bc-ad)^2(a+bx)^{3/4}} + \\
 & \left( \frac{4\sqrt{2}d^{7/4}((a+bx)(c+dx))^{3/4}\sqrt{(bc+ad+2bdx)^2}}{\left(1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)\sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2\left(1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad}\right)^2}}}\right) \\
 & \left. \text{EllipticF}\left[2\text{ArcTan}\left[\frac{\sqrt{2}b^{1/4}d^{1/4}((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}}\right], \frac{1}{2}\right] \right) / \\
 & \left(7b^{1/4}(bc-ad)^{3/2}(a+bx)^{3/4}(c+dx)^{3/4}(bc+ad+2bdx)\sqrt{(ad+b(c+2dx))^2}\right)
 \end{aligned}$$

Result (type 5, 102 leaves):

$$\begin{aligned}
 & \left(4(c+dx)^{1/4}\left(-bc+3ad+2bdx+4d(a+bx)\left(\frac{d(a+bx)}{-bc+ad}\right)^{3/4}\right.\right. \\
 & \left.\left.\text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{b(c+dx)}{bc-ad}\right]\right)\right) / \left(7(bc-ad)^2(a+bx)^{7/4}\right)
 \end{aligned}$$

**Problem 1717: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{5/4}}{(c+dx)^{5/4}} dx$$

Optimal (type 3, 152 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{4(a+bx)^{5/4}}{d(c+dx)^{1/4}} + \frac{5b(a+bx)^{1/4}(c+dx)^{3/4}}{d^2} - \\
 & \frac{5b^{1/4}(bc-ad)\text{ArcTan}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{2d^{9/4}} - \frac{5b^{1/4}(bc-ad)\text{ArcTanh}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{2d^{9/4}}
 \end{aligned}$$

Result (type 5, 99 leaves):

$$\frac{1}{3d^2}(a+bx)^{1/4}(c+dx)^{3/4}\left(\frac{3(5bc-4ad+bdx)}{c+dx} + \frac{5b\text{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right]}{\left(\frac{d(a+bx)}{-bc+ad}\right)^{1/4}}\right)$$

**Problem 1718: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/4}}{(c+dx)^{5/4}} dx$$

Optimal (type 3, 108 leaves, 6 steps):

$$-\frac{4(a+bx)^{1/4}}{d(c+dx)^{1/4}} + \frac{2b^{1/4} \operatorname{ArcTan}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{d^{5/4}} + \frac{2b^{1/4} \operatorname{ArcTanh}\left[\frac{d^{1/4}(a+bx)^{1/4}}{b^{1/4}(c+dx)^{1/4}}\right]}{d^{5/4}}$$

Result (type 5, 89 leaves):

$$\frac{4 \left( -3d(a+bx) + b \left( \frac{d(a+bx)}{-bc+ad} \right)^{3/4} (c+dx) \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad}\right] \right)}{(3d^2(a+bx)^{3/4}(c+dx)^{1/4})}$$

**Problem 1723: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{11/4}}{(c+dx)^{5/4}} dx$$

Optimal (type 4, 776 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{4 (a+bx)^{11/4}}{d (c+dx)^{1/4}} - \frac{77 b (bc-ad) (a+bx)^{3/4} (c+dx)^{3/4}}{15 d^3} + \frac{22 b (a+bx)^{7/4} (c+dx)^{3/4}}{5 d^2} + \\
 & \left( 77 \sqrt{b} (bc-ad) \sqrt{(a+bx)(c+dx)} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2} \right) / \\
 & \left( 10 d^{7/2} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \right) - \\
 & \left( 77 b^{1/4} (bc-ad)^{7/2} ((a+bx)(c+dx))^{1/4} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( 10 \sqrt{2} d^{15/4} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right) + \\
 & \left( 77 b^{1/4} (bc-ad)^{7/2} ((a+bx)(c+dx))^{1/4} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( 20 \sqrt{2} d^{15/4} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 132 leaves):



$$\frac{1}{15 d^4 (a + b x)^{1/4}} (c + d x)^{3/4} \left( d (a + b x) \left( b (-17 b c + 23 a d) + 6 b^2 d x - \frac{60 (b c - a d)^2}{c + d x} \right) + 77 b (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/4} \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

**Problem 1724: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{7/4}}{(c + d x)^{5/4}} dx$$

Optimal (type 4, 730 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{4 (a+bx)^{7/4}}{d (c+dx)^{1/4}} + \frac{14 b (a+bx)^{3/4} (c+dx)^{3/4}}{3 d^2} - \\
 & \left( 7 \sqrt{b} \sqrt{(a+bx)(c+dx)} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2} \right) / \\
 & \left( d^{5/2} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \right) + \\
 & \left( 7 b^{1/4} (bc-ad)^{5/2} ((a+bx)(c+dx))^{1/4} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( \sqrt{2} d^{11/4} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right) - \\
 & \left( 7 b^{1/4} (bc-ad)^{5/2} ((a+bx)(c+dx))^{1/4} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( 2 \sqrt{2} d^{11/4} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 98 leaves):

$$\frac{1}{3 d^2} (a + b x)^{3/4} (c + d x)^{3/4} \left( \frac{7 b c - 6 a d + b d x}{c + d x} + \frac{7 b \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{b c - a d}\right]}{\left(\frac{d(a+bx)}{-b c + a d}\right)^{3/4}} \right)$$

Problem 1725: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{3/4}}{(c + d x)^{5/4}} dx$$

Optimal (type 4, 712 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{4(a+bx)^{3/4}}{d(c+dx)^{1/4}} + \left( 6\sqrt{b}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2} \right) / \\
 & \left( d^{3/2}(bc-ad)(a+bx)^{1/4}(c+dx)^{1/4}(bc+ad+2bdx) \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \right) - \\
 & \left( 3\sqrt{2}b^{1/4}(bc-ad)^{3/2}((a+bx)(c+dx))^{1/4}\sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2}b^{1/4}d^{1/4}((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( d^{7/4}(a+bx)^{1/4}(c+dx)^{1/4}(bc+ad+2bdx)\sqrt{(ad+b(c+2dx))^2} \right) + \\
 & \left( 3b^{1/4}(bc-ad)^{3/2}((a+bx)(c+dx))^{1/4}\sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2}b^{1/4}d^{1/4}((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( \sqrt{2}d^{7/4}(a+bx)^{1/4}(c+dx)^{1/4}(bc+ad+2bdx)\sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 87 leaves):

$$\begin{aligned}
 & \left( -4d(a+bx) + 4b \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/4} (c+dx) \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad} \right] \right) / \\
 & \left( d^2(a+bx)^{1/4}(c+dx)^{1/4} \right)
 \end{aligned}$$

Problem 1726: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{1/4} (c + d x)^{5/4}} dx$$

Optimal (type 4, 719 leaves, 6 steps):

$$\begin{aligned}
 & \frac{4 (a+bx)^{3/4}}{(bc-ad)(c+dx)^{1/4}} - \left( 4\sqrt{b} \sqrt{(a+bx)(c+dx)} \sqrt{(bc+ad+2bdx)^2} \sqrt{(ad+b(c+2dx))^2} \right) / \\
 & \left( \sqrt{d} (bc-ad)^2 (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \left( 1 + \frac{2\sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \right) + \\
 & \left( 2\sqrt{2} b^{1/4} \sqrt{bc-ad} ((a+bx)(c+dx))^{1/4} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2\sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( d^{3/4} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right) - \\
 & \left( \sqrt{2} b^{1/4} \sqrt{bc-ad} ((a+bx)(c+dx))^{1/4} \sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2\sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b} \sqrt{d} \sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( d^{3/4} (a+bx)^{1/4} (c+dx)^{1/4} (bc+ad+2bdx) \sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 100 leaves):

$$\begin{aligned}
 & \left( 12d(a+bx) - 8b \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/4} (c+dx) \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b(c+dx)}{bc-ad} \right] \right) / \\
 & \left( 3d(bc-ad)(a+bx)^{1/4}(c+dx)^{1/4} \right)
 \end{aligned}$$

Problem 1727: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a + b x)^{5/4} (c + d x)^{5/4}} dx$$

Optimal (type 4, 750 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{4}{(bc-ad)(a+bx)^{1/4}(c+dx)^{1/4}} - \frac{8d(a+bx)^{3/4}}{(bc-ad)^2(c+dx)^{1/4}} + \\
 & \left( 8\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}\sqrt{(bc+ad+2bdx)^2}\sqrt{(ad+b(c+2dx))^2} \right) / \\
 & \left( (bc-ad)^3(a+bx)^{1/4}(c+dx)^{1/4}(bc+ad+2bdx) \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \right) - \\
 & \left( 4\sqrt{2}b^{1/4}d^{1/4}((a+bx)(c+dx))^{1/4}\sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2}b^{1/4}d^{1/4}((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( \sqrt{bc-ad}(a+bx)^{1/4}(c+dx)^{1/4}(bc+ad+2bdx)\sqrt{(ad+b(c+2dx))^2} \right) + \\
 & \left( 2\sqrt{2}b^{1/4}d^{1/4}((a+bx)(c+dx))^{1/4}\sqrt{(bc+ad+2bdx)^2} \right. \\
 & \left. \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right) \sqrt{\frac{(ad+b(c+2dx))^2}{(bc-ad)^2 \left( 1 + \frac{2\sqrt{b}\sqrt{d}\sqrt{(a+bx)(c+dx)}}{bc-ad} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2}b^{1/4}d^{1/4}((a+bx)(c+dx))^{1/4}}{\sqrt{bc-ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( \sqrt{bc-ad}(a+bx)^{1/4}(c+dx)^{1/4}(bc+ad+2bdx)\sqrt{(ad+b(c+2dx))^2} \right)
 \end{aligned}$$

Result (type 5, 102 leaves):



$$\begin{aligned}
 & - \left( \left( 4 \left( 3 a d + 3 b (c + 2 d x) - 4 b \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/4} (c + d x) \right. \right. \right. \\
 & \quad \left. \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d} \right] \right) \right) \right) / \left( 3 (b c - a d)^2 (a + b x)^{1/4} (c + d x)^{1/4} \right)
 \end{aligned}$$

**Problem 1728: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b x)^{9/4} (c + d x)^{5/4}} dx$$

Optimal (type 4, 795 leaves, 8 steps):

$$\begin{aligned}
 & - \frac{4}{5 (bc - ad) (a + bx)^{5/4} (c + dx)^{1/4}} + \\
 & \frac{24 d}{5 (bc - ad)^2 (a + bx)^{1/4} (c + dx)^{1/4}} + \frac{48 d^2 (a + bx)^{3/4}}{5 (bc - ad)^3 (c + dx)^{1/4}} - \\
 & \left( \frac{48 \sqrt{b} d^{3/2} \sqrt{(a + bx) (c + dx)} \sqrt{(bc + ad + 2bdx)^2} \sqrt{(ad + b(c + 2dx))^2}}{5 (bc - ad)^4 (a + bx)^{1/4} (c + dx)^{1/4} (bc + ad + 2bdx)} \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right) \right) + \\
 & \left( \frac{24 \sqrt{2} b^{1/4} d^{5/4} ((a + bx) (c + dx))^{1/4} \sqrt{(bc + ad + 2bdx)^2}}{\left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right) \sqrt{\frac{(ad + b(c + 2dx))^2}{(bc - ad)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right)^2}} \right) \\
 & \left. \text{EllipticE} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + bx) (c + dx))^{1/4}}{\sqrt{bc - ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( 5 (bc - ad)^{3/2} (a + bx)^{1/4} (c + dx)^{1/4} (bc + ad + 2bdx) \sqrt{(ad + b(c + 2dx))^2} \right) - \\
 & \left( \frac{12 \sqrt{2} b^{1/4} d^{5/4} ((a + bx) (c + dx))^{1/4} \sqrt{(bc + ad + 2bdx)^2}}{\left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right) \sqrt{\frac{(ad + b(c + 2dx))^2}{(bc - ad)^2 \left( 1 + \frac{2 \sqrt{b} \sqrt{d} \sqrt{(a + bx) (c + dx)}}{bc - ad} \right)^2}} \right) \\
 & \left. \text{EllipticF} \left[ 2 \text{ArcTan} \left[ \frac{\sqrt{2} b^{1/4} d^{1/4} ((a + bx) (c + dx))^{1/4}}{\sqrt{bc - ad}} \right], \frac{1}{2} \right] \right) / \\
 & \left( 5 (bc - ad)^{3/2} (a + bx)^{1/4} (c + dx)^{1/4} (bc + ad + 2bdx) \sqrt{(ad + b(c + 2dx))^2} \right)
 \end{aligned}$$

Result(type 5, 139 leaves):

$$\begin{aligned}
 & - \left( \left( 4 \left( 5 a^2 d^2 + 2 a b d (4 c + 9 d x) + b^2 (-c^2 + 6 c d x + 12 d^2 x^2) - \right. \right. \right. \\
 & \quad \left. \left. \left. 8 b d (a + b x) \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/4} (c + d x) \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{3}{4}, \frac{7}{4}, \frac{b (c + d x)}{b c - a d} \right] \right) \right) / \\
 & \quad \left( 5 (-b c + a d)^3 (a + b x)^{5/4} (c + d x)^{1/4} \right)
 \end{aligned}$$

**Problem 1729: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(1 - a x)^{1/4} (1 + b x)^{3/4}} dx$$

Optimal (type 3, 279 leaves, 11 steps):

$$\begin{aligned}
 & \frac{\sqrt{2} \operatorname{ArcTan} \left[ 1 - \frac{\sqrt{2} b^{1/4} (1 - a x)^{1/4}}{a^{1/4} (1 + b x)^{1/4}} \right]}{a^{1/4} b^{3/4}} - \frac{\sqrt{2} \operatorname{ArcTan} \left[ 1 + \frac{\sqrt{2} b^{1/4} (1 - a x)^{1/4}}{a^{1/4} (1 + b x)^{1/4}} \right]}{a^{1/4} b^{3/4}} - \\
 & \frac{\operatorname{Log} \left[ \sqrt{a} + \frac{\sqrt{b} \sqrt{1 - a x}}{\sqrt{1 + b x}} - \frac{\sqrt{2} a^{1/4} b^{1/4} (1 - a x)^{1/4}}{(1 + b x)^{1/4}} \right]}{\sqrt{2} a^{1/4} b^{3/4}} + \frac{\operatorname{Log} \left[ \sqrt{a} + \frac{\sqrt{b} \sqrt{1 - a x}}{\sqrt{1 + b x}} + \frac{\sqrt{2} a^{1/4} b^{1/4} (1 - a x)^{1/4}}{(1 + b x)^{1/4}} \right]}{\sqrt{2} a^{1/4} b^{3/4}}
 \end{aligned}$$

Result (type 5, 63 leaves):

$$\frac{4 (1 + b x)^{1/4} \left( \frac{b - a b x}{a + b} \right)^{1/4} \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{a + a b x}{a + b} \right]}{b (1 - a x)^{1/4}}$$

**Problem 1730: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(1 - a x)^{1/4} (1 + a x)^{3/4}} dx$$

Optimal (type 3, 193 leaves, 11 steps):

$$\begin{aligned}
 & \frac{\sqrt{2} \operatorname{ArcTan} \left[ 1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{a} - \frac{\sqrt{2} \operatorname{ArcTan} \left[ 1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{a} - \\
 & \frac{\operatorname{Log} \left[ 1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{\sqrt{2} a} + \frac{\operatorname{Log} \left[ 1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{\sqrt{2} a}
 \end{aligned}$$

Result (type 5, 38 leaves):

$$\frac{2 \times 2^{3/4} (1 + a x)^{1/4} \operatorname{Hypergeometric2F1} \left[ \frac{1}{4}, \frac{1}{4}, \frac{5}{4}, \frac{1}{2} (1 + a x) \right]}{a}$$

**Problem 1736: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^{5/2} (c + d x)^{1/6} dx$$

Optimal (type 4, 487 leaves, 6 steps):

$$\frac{81 (b c - a d)^3 \sqrt{a + b x} (c + d x)^{1/6}}{1408 b d^3} - \frac{9 (b c - a d)^2 (a + b x)^{3/2} (c + d x)^{1/6}}{352 b d^2} +$$

$$\frac{3 (b c - a d) (a + b x)^{5/2} (c + d x)^{1/6}}{176 b d} + \frac{3 (a + b x)^{7/2} (c + d x)^{1/6}}{11 b} -$$

$$\left( 81 \times 3^{3/4} (b c - a d)^{11/3} (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \right.$$

$$\sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}}$$

$$\text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}}, \frac{1}{4} (2 + \sqrt{3}) \right] \right] /$$

$$\left( 2816 b d^4 \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right)$$

Result (type 5, 181 leaves):

$$- \left( \left( 3 (c + d x)^{1/6} \right. \right.$$

$$\left. \left( -d (a + b x) (81 a^3 d^3 + a^2 b d^2 (113 c + 356 d x) + a b^2 d (-93 c^2 + 40 c d x + 376 d^2 x^2) + \right. \right.$$

$$\left. \left. b^3 (27 c^3 - 12 c^2 d x + 8 c d^2 x^2 + 128 d^3 x^3) \right) + 81 (b c - a d)^4 \sqrt{\frac{d (a + b x)}{-b c + a d}} \right.$$

$$\left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d} \right] \right) \right) / (1408 b d^4 \sqrt{a + b x})$$

**Problem 1737: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^{3/2} (c + d x)^{1/6} dx$$

Optimal (type 4, 449 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{27 (bc - ad)^2 \sqrt{a + bx} (c + dx)^{1/6}}{320 b d^2} + \frac{3 (bc - ad) (a + bx)^{3/2} (c + dx)^{1/6}}{80 b d} + \\
 & \frac{3 (a + bx)^{5/2} (c + dx)^{1/6}}{8 b} + \left( 27 \times 3^{3/4} (bc - ad)^{8/3} (c + dx)^{1/6} \left( (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right) \right. \\
 & \sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c + dx)^{1/3} + b^{2/3} (c + dx)^{2/3}}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3} \right)^2}} \\
 & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc - ad)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + dx)^{1/3}}{(bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\
 & \left( 640 b d^3 \sqrt{a + bx} \sqrt{-\frac{b^{1/3} (c + dx)^{1/3} \left( (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right)}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3} \right)^2}} \right)
 \end{aligned}$$

Result (type 5, 142 leaves):

$$\begin{aligned}
 & -\frac{1}{320 b d^3 \sqrt{a + bx}} \\
 & 3 (c + dx)^{1/6} \left( -d (a + bx) (27 a^2 d^2 + 2 a b d (11 c + 38 d x) + b^2 (-9 c^2 + 4 c d x + 40 d^2 x^2)) - \right. \\
 & \left. 27 (bc - ad)^3 \sqrt{\frac{d (a + bx)}{-bc + ad}} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c + dx)}{bc - ad} \right] \right)
 \end{aligned}$$

**Problem 1738: Result unnecessarily involves higher level functions.**

$$\int \sqrt{a + bx} (c + dx)^{1/6} dx$$

Optimal (type 4, 411 leaves, 4 steps):

$$\frac{3 (b c - a d) \sqrt{a + b x} (c + d x)^{1/6}}{20 b d} + \frac{3 (a + b x)^{3/2} (c + d x)^{1/6}}{5 b} -$$

$$\left( 3 \times 3^{3/4} (b c - a d)^{5/3} (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \right.$$

$$\sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}}$$

$$\text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \Bigg/$$

$$\left( 40 b d^2 \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right)$$

Result (type 5, 109 leaves):

$$\frac{1}{20 b d^2 \sqrt{a + b x}} 3 (c + d x)^{1/6} \left( d (a + b x) (3 a d + b (c + 4 d x)) - \right.$$

$$\left. 3 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d} \right] \right)$$

**Problem 1739: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + d x)^{1/6}}{\sqrt{a + b x}} dx$$

Optimal (type 4, 375 leaves, 3 steps):

$$\frac{3 \sqrt{a+bx} (c+dx)^{1/6}}{2b} + \left( 3^{3/4} (bc-ad)^{2/3} (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\ \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\ \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] \right) / \\ \left( 4bd \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right)$$

Result (type 5, 93 leaves):

$$\frac{1}{2bd \sqrt{a+bx}} \\ 3 (c+dx)^{1/6} \left( d(a+bx) + (bc-ad) \sqrt{\frac{d(a+bx)}{-bc+ad}} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad} \right] \right)$$

**Problem 1740: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{1/6}}{(a+bx)^{3/2}} dx$$

Optimal (type 4, 367 leaves, 3 steps):

$$-\frac{2(c+dx)^{1/6}}{b \sqrt{a+bx}} + \left( (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\ \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\ \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] \right) / \\ \left( 3^{1/4} b (bc-ad)^{1/3} \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right)$$

Result (type 5, 74 leaves):

$$\frac{2 (c + d x)^{1/6} \left( -1 + \sqrt{\frac{d (a + b x)}{-b c + a d}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d} \right] \right)}{b \sqrt{a + b x}}$$

**Problem 1741: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + d x)^{1/6}}{(a + b x)^{5/2}} dx$$

Optimal (type 4, 409 leaves, 4 steps):

$$\begin{aligned} & -\frac{2 (c + d x)^{1/6}}{3 b (a + b x)^{3/2}} - \frac{2 d (c + d x)^{1/6}}{9 b (b c - a d) \sqrt{a + b x}} - \left( 2 d (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \right. \\ & \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\ & \left. \operatorname{EllipticF} \left[ \operatorname{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\ & \left( 9 \times 3^{1/4} b (b c - a d)^{4/3} \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right) \end{aligned}$$

Result (type 5, 104 leaves):

$$\begin{aligned} & \left( 2 (c + d x)^{1/6} \right. \\ & \left. \left( 3 b c - 2 a d + b d x + 2 d (a + b x) \sqrt{\frac{d (a + b x)}{-b c + a d}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d} \right] \right) \right) / \\ & (9 b (-b c + a d) (a + b x)^{3/2}) \end{aligned}$$

**Problem 1742: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^{3/2} (c + d x)^{5/6} dx$$

Optimal (type 4, 896 leaves, 7 steps):



$$\begin{aligned}
 & - \frac{27 (bc - ad)^2 \sqrt{a+bx} (c+dx)^{5/6}}{224 b d^2} + \frac{3 (bc - ad) (a+bx)^{3/2} (c+dx)^{5/6}}{28 b d} + \\
 & \frac{3 (a+bx)^{5/2} (c+dx)^{5/6}}{10 b} - \frac{81 (1 + \sqrt{3}) (bc - ad)^3 \sqrt{a+bx} (c+dx)^{1/6}}{448 b^{5/3} d^2 \left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)} - \\
 & \left( \frac{81 \times 3^{1/4} (bc - ad)^{10/3} (c+dx)^{1/6} \left( (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}}} \right. \\
 & \left. \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(bc - ad)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c+dx)^{1/3}}, \frac{1}{4} (2 + \sqrt{3}) \right] \right] \right) / \\
 & \left( 448 b^{5/3} d^3 \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right) - \\
 & \left( 27 \times 3^{3/4} (1 - \sqrt{3}) (bc - ad)^{10/3} (c+dx)^{1/6} \left( (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\
 & \left. \sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc - ad)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c+dx)^{1/3}}, \frac{1}{4} (2 + \sqrt{3}) \right] \right] \right) / \\
 & \left( 896 b^{5/3} d^3 \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc - ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right)
 \end{aligned}$$

Result (type 5, 142 leaves):

$$- \left( \left( 3 (c + dx)^{5/6} \right. \right. \\ \left. \left. \left( -d (a + bx) (27 a^2 d^2 + 2 a b d (65 c + 92 dx) + b^2 (-45 c^2 + 40 c dx + 112 d^2 x^2)) - 27 (bc - ad)^3 \right. \right. \right. \\ \left. \left. \left. \sqrt{\frac{d(a + bx)}{-bc + ad}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b(c + dx)}{bc - ad} \right] \right) \right) / (1120 b d^3 \sqrt{a + bx}) \right)$$

**Problem 1743: Result unnecessarily involves higher level functions.**

$$\int \sqrt{a + bx} (c + dx)^{5/6} dx$$

Optimal (type 4, 858 leaves, 6 steps):

$$\begin{aligned}
 & \frac{15 (b c - a d) \sqrt{a + b x} (c + d x)^{5/6}}{56 b d} + \frac{3 (a + b x)^{3/2} (c + d x)^{5/6}}{7 b} + \\
 & \frac{45 (1 + \sqrt{3}) (b c - a d)^2 \sqrt{a + b x} (c + d x)^{1/6}}{112 b^{5/3} d \left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)} + \\
 & \left( 45 \times 3^{1/4} (b c - a d)^{7/3} (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \right. \\
 & \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\
 & \left( 112 b^{5/3} d^2 \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right) + \\
 & \left( 15 \times 3^{3/4} (1 - \sqrt{3}) (b c - a d)^{7/3} (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \right. \\
 & \left. \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\
 & \left( 224 b^{5/3} d^2 \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
 \end{aligned}$$

Result (type 5, 110 leaves):

$$\begin{aligned}
 & \frac{1}{56 b d^2 \sqrt{a + b x}} 3 (c + d x)^{5/6} \left( d (a + b x) (5 b c + 3 a d + 8 b d x) - \right. \\
 & \left. 3 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d} \right] \right)
 \end{aligned}$$

**Problem 1744: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{5/6}}{\sqrt{a+bx}} dx$$

Optimal (type 4, 817 leaves, 5 steps):

$$\frac{3\sqrt{a+bx}(c+dx)^{5/6}}{4b} - \frac{15(1+\sqrt{3})(bc-ad)\sqrt{a+bx}(c+dx)^{1/6}}{8b^{5/3}\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)} -$$

$$\left( \frac{15 \times 3^{1/4} (bc-ad)^{4/3} (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}}}$$

$$\text{EllipticE}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right) /$$

$$\left( 8b^{5/3} d \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right) -$$

$$\left( 5 \times 3^{3/4} (1-\sqrt{3}) (bc-ad)^{4/3} (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right.$$

$$\left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right.$$

$$\left. \text{EllipticF}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right) /$$

$$\left( 16b^{5/3} d \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right)$$

Result (type 5, 93 leaves):

$$\frac{1}{4 b d \sqrt{a+b x}} \int \frac{(c+d x)^{5/6}}{(a+b x)^{3/2}} d x = \frac{d (a+b x) + (b c-a d) \sqrt{\frac{d (a+b x)}{-b c+a d}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c+d x)}{b c-a d}\right]}{3 (c+d x)^{5/6}}$$

**Problem 1745: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+d x)^{5/6}}{(a+b x)^{3/2}} d x$$

Optimal (type 4, 798 leaves, 5 steps):

$$\begin{aligned} & \frac{2 (c+d x)^{5/6}}{b \sqrt{a+b x}} - \frac{5 (1+\sqrt{3}) d \sqrt{a+b x} (c+d x)^{1/6}}{b^{5/3} \left( (b c-a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3} \right)} \\ & \left( \frac{5 \times 3^{1/4} (b c-a d)^{1/3} (c+d x)^{1/6} \left( (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)}{\sqrt{\frac{(b c-a d)^{2/3} + b^{1/3} (b c-a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{\left( (b c-a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3} \right)^2}}} \right. \\ & \left. \operatorname{EllipticE}\left[\operatorname{ArcCos}\left[\frac{(b c-a d)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+d x)^{1/3}}{(b c-a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3}}\right], \frac{1}{4} (2+\sqrt{3})\right] \right) / \\ & \left( b^{5/3} \sqrt{a+b x} \sqrt{-\frac{b^{1/3} (c+d x)^{1/3} \left( (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)}{\left( (b c-a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3} \right)^2}} \right) - \\ & \left( \frac{5 (1-\sqrt{3}) (b c-a d)^{1/3} (c+d x)^{1/6} \left( (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)}{\sqrt{\frac{(b c-a d)^{2/3} + b^{1/3} (b c-a d)^{1/3} (c+d x)^{1/3} + b^{2/3} (c+d x)^{2/3}}{\left( (b c-a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3} \right)^2}}} \right. \\ & \left. \operatorname{EllipticF}\left[\operatorname{ArcCos}\left[\frac{(b c-a d)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+d x)^{1/3}}{(b c-a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3}}\right], \frac{1}{4} (2+\sqrt{3})\right] \right) / \\ & \left( 2 \times 3^{1/4} b^{5/3} \sqrt{a+b x} \sqrt{-\frac{b^{1/3} (c+d x)^{1/3} \left( (b c-a d)^{1/3} - b^{1/3} (c+d x)^{1/3} \right)}{\left( (b c-a d)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+d x)^{1/3} \right)^2}} \right) \end{aligned}$$

Result (type 5, 74 leaves):

$$\frac{2 (c + d x)^{5/6} \left( -1 + \sqrt{\frac{d (a + b x)}{-b c + a d}} \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d} \right] \right)}{b \sqrt{a + b x}}$$

**Problem 1746: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + d x)^{5/6}}{(a + b x)^{5/2}} dx$$

Optimal (type 4, 854 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{2(c+dx)^{5/6}}{3b(a+bx)^{3/2}} - \frac{10d(c+dx)^{5/6}}{9b(bc-ad)\sqrt{a+bx}} - \\
 & \frac{10(1+\sqrt{3})d^2\sqrt{a+bx}(c+dx)^{1/6}}{9b^{5/3}(bc-ad)\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)} - \\
 & \left(10d(c+dx)^{1/6}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)\right. \\
 & \left.\sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}}\right. \\
 & \left.\text{EllipticE}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right]\right) / \\
 & \left(3 \times 3^{3/4} b^{5/3} (bc-ad)^{2/3} \sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}}\right) - \\
 & \left(5(1-\sqrt{3})d(c+dx)^{1/6}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)\right. \\
 & \left.\sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}}\right. \\
 & \left.\text{EllipticF}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right]\right) / \\
 & \left(9 \times 3^{1/4} b^{5/3} (bc-ad)^{2/3} \sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}}\right)
 \end{aligned}$$

Result (type 5, 105 leaves):

$$\begin{aligned}
 & -\left(\left(2(c+dx)^{5/6}\left(3bc+2ad+5bdx-2d(a+bx)\sqrt{\frac{d(a+bx)}{-bc+ad}}\right.\right.\right. \\
 & \left.\left.\left.\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad}\right]\right)\right)\right) / \left(9b(bc-ad)(a+bx)^{3/2}\right)
 \end{aligned}$$

**Problem 1747: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{7/2}} dx$$

Optimal (type 4, 896 leaves, 7 steps):

$$\begin{aligned} & -\frac{2(c+dx)^{5/6}}{5b(a+bx)^{5/2}} - \frac{2d(c+dx)^{5/6}}{9b(bc-ad)(a+bx)^{3/2}} + \frac{8d^2(c+dx)^{5/6}}{27b(bc-ad)^2\sqrt{a+bx}} + \\ & \frac{8(1+\sqrt{3})d^3\sqrt{a+bx}(c+dx)^{1/6}}{27b^{5/3}(bc-ad)^2((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3})} + \\ & \left( 8d^2(c+dx)^{1/6}((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}) \right. \\ & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3})^2}} \right. \\ & \left. \text{EllipticE}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right) / \\ & \left( 9 \times 3^{3/4} b^{5/3} (bc-ad)^{5/3} \sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3}((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3})}{((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3})^2}} \right) + \\ & \left( 4(1-\sqrt{3})d^2(c+dx)^{1/6}((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}) \right. \\ & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3})^2}} \right. \\ & \left. \text{EllipticF}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right) / \\ & \left( 27 \times 3^{1/4} b^{5/3} (bc-ad)^{5/3} \sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3}((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3})}{((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3})^2}} \right) \end{aligned}$$

Result (type 5, 140 leaves):



$$\begin{aligned}
 & - \left( \left( 2 (c + dx)^{5/6} \right. \right. \\
 & \quad \left. \left. \left( -8 a^2 d^2 - a b d (39 c + 55 dx) + b^2 (27 c^2 + 15 c dx - 20 d^2 x^2) + 8 d^2 (a + bx)^2 \sqrt{\frac{d(a + bx)}{-bc + ad}} \right. \right. \right. \\
 & \quad \left. \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b(c + dx)}{bc - ad} \right] \right) \right) / \left( 135 b (bc - ad)^2 (a + bx)^{5/2} \right)
 \end{aligned}$$

**Problem 1748: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + bx)^{5/2}}{(c + dx)^{1/6}} dx$$

Optimal (type 4, 890 leaves, 7 steps):

$$\frac{81 (bc - ad)^2 \sqrt{a + bx} (c + dx)^{5/6}}{224 d^3} - \frac{9 (bc - ad) (a + bx)^{3/2} (c + dx)^{5/6}}{28 d^2} +$$

$$\frac{3 (a + bx)^{5/2} (c + dx)^{5/6}}{10 d} + \frac{243 (1 + \sqrt{3}) (bc - ad)^3 \sqrt{a + bx} (c + dx)^{1/6}}{448 b^{2/3} d^3 \left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3} \right)} +$$

$$\left( \frac{243 \times 3^{1/4} (bc - ad)^{10/3} (c + dx)^{1/6} \left( (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right)}{\sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c + dx)^{1/3} + b^{2/3} (c + dx)^{2/3}}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3} \right)^2}}}$$

$$\text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(bc - ad)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + dx)^{1/3}}{(bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3}}, \frac{1}{4} (2 + \sqrt{3}) \right] \right] /$$

$$\left( 448 b^{2/3} d^4 \sqrt{a + bx} \sqrt{-\frac{b^{1/3} (c + dx)^{1/3} \left( (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right)}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3} \right)^2}} \right) +$$

$$\left( 81 \times 3^{3/4} (1 - \sqrt{3}) (bc - ad)^{10/3} (c + dx)^{1/6} \left( (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right) \right.$$

$$\left. \sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c + dx)^{1/3} + b^{2/3} (c + dx)^{2/3}}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3} \right)^2}} \right.$$

$$\left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc - ad)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + dx)^{1/3}}{(bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3}}, \frac{1}{4} (2 + \sqrt{3}) \right] \right] / \right.$$

$$\left( 896 b^{2/3} d^4 \sqrt{a + bx} \sqrt{-\frac{b^{1/3} (c + dx)^{1/3} \left( (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right)}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3} \right)^2}} \right)$$

Result (type 5, 138 leaves):

$$\frac{1}{1120 d^4 \sqrt{a + bx}}$$

$$3 (c + dx)^{5/6} \left( d (a + bx) (367 a^2 d^2 + 2 a b d (-195 c + 172 dx) + b^2 (135 c^2 - 120 c dx + 112 d^2 x^2)) - \right.$$

$$\left. 81 (bc - ad)^3 \sqrt{\frac{d (a + bx)}{-bc + ad}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + dx)}{bc - ad} \right] \right)$$

### Problem 1749: Result unnecessarily involves higher level functions.

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{1/6}} dx$$

Optimal (type 4, 855 leaves, 6 steps):

$$\begin{aligned} & -\frac{27(bc-ad)\sqrt{a+bx}(c+dx)^{5/6}}{56d^2} + \frac{3(a+bx)^{3/2}(c+dx)^{5/6}}{7d} - \\ & \frac{81(1+\sqrt{3})(bc-ad)^2\sqrt{a+bx}(c+dx)^{1/6}}{112b^{2/3}d^2\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)} - \\ & \left( 81 \times 3^{1/4} (bc-ad)^{7/3} (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\ & \quad \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}} \\ & \quad \left. \text{EllipticE}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right) / \\ & \left( 112b^{2/3}d^3\sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}} \right) - \\ & \left( 27 \times 3^{3/4} (1-\sqrt{3})(bc-ad)^{7/3}(c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right) \right. \\ & \quad \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}} \\ & \quad \left. \text{EllipticF}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right) / \\ & \left( 224b^{2/3}d^3\sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}} \right) \end{aligned}$$

Result (type 5, 108 leaves):

$$\frac{1}{280 d^3 \sqrt{a + b x}} 3 (c + d x)^{5/6} \left( 5 d (a + b x) (-9 b c + 17 a d + 8 b d x) + \right. \\ \left. 27 (b c - a d)^2 \sqrt{\frac{d (a + b x)}{-b c + a d}} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d}\right] \right)$$

Problem 1750: Result unnecessarily involves higher level functions.

$$\int \frac{\sqrt{a + b x}}{(c + d x)^{1/6}} dx$$

Optimal (type 4, 820 leaves, 5 steps):

$$\begin{aligned}
 & \frac{3\sqrt{a+bx}(c+dx)^{5/6}}{4d} + \frac{9(1+\sqrt{3})(bc-ad)\sqrt{a+bx}(c+dx)^{1/6}}{8b^{2/3}d\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)} + \\
 & \left( \frac{9 \times 3^{1/4}(bc-ad)^{4/3}(c+dx)^{1/6}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)}{\sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}}} \right. \\
 & \left. \text{EllipticE}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right]\right) / \\
 & \left( 8b^{2/3}d^2\sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}} \right) + \\
 & \left( 3 \times 3^{3/4}(1-\sqrt{3})(bc-ad)^{4/3}(c+dx)^{1/6}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right) \right. \\
 & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}} \right. \\
 & \left. \text{EllipticF}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right]\right) / \\
 & \left( 16b^{2/3}d^2\sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}} \right)
 \end{aligned}$$

Result (type 5, 77 leaves):

$$\frac{3\sqrt{a+bx}(c+dx)^{5/6} \left( 5 + \frac{{}_3\text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad}\right]}{\sqrt{\frac{d(a-bx)}{-bc+ad}}} \right)}{20d}$$

**Problem 1751: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{a+bx}(c+dx)^{1/6}} dx$$

Optimal (type 4, 780 leaves, 4 steps):

$$\begin{aligned}
 & - \frac{3 (1 + \sqrt{3}) \sqrt{a + b x} (c + d x)^{1/6}}{b^{2/3} \left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)} \\
 & \left( 3 \times 3^{1/4} (b c - a d)^{1/3} (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \right. \\
 & \quad \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \\
 & \quad \left. \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\
 & \left( b^{2/3} d \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right) - \\
 & \left( 3^{3/4} (1 - \sqrt{3}) (b c - a d)^{1/3} (c + d x)^{1/6} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right) \right. \\
 & \quad \sqrt{\frac{(b c - a d)^{2/3} + b^{1/3} (b c - a d)^{1/3} (c + d x)^{1/3} + b^{2/3} (c + d x)^{2/3}}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \\
 & \quad \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(b c - a d)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + d x)^{1/3}}{(b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\
 & \left( 2 b^{2/3} d \sqrt{a + b x} \sqrt{-\frac{b^{1/3} (c + d x)^{1/3} \left( (b c - a d)^{1/3} - b^{1/3} (c + d x)^{1/3} \right)}{\left( (b c - a d)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + d x)^{1/3} \right)^2}} \right)
 \end{aligned}$$

Result (type 5, 73 leaves):

$$\frac{6 \sqrt{\frac{d(a+bx)}{-bc+ad}} (c+dx)^{5/6} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{b c - a d} \right]}{5 d \sqrt{a + b x}}$$

**Problem 1752: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b x)^{3/2} (c + d x)^{1/6}} dx$$

Optimal (type 4, 813 leaves, 5 steps):

$$\begin{aligned}
 & - \frac{2 (c+dx)^{5/6}}{(bc-ad) \sqrt{a+bx}} - \frac{2 (1+\sqrt{3}) d \sqrt{a+bx} (c+dx)^{1/6}}{b^{2/3} (bc-ad) \left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)} - \\
 & \left( 2 \times 3^{1/4} (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\
 & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] \right) / \\
 & \left( b^{2/3} (bc-ad)^{2/3} \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right) - \\
 & \left( (1-\sqrt{3}) (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\
 & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] \right) / \\
 & \left( 3^{1/4} b^{2/3} (bc-ad)^{2/3} \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right)
 \end{aligned}$$

Result (type 5, 84 leaves):

$$\left( 2 (c+dx)^{5/6} \left( -5 + 2 \sqrt{\frac{d(a+bx)}{-bc+ad}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad} \right] \right) \right) / (5 (bc-ad) \sqrt{a+bx})$$

**Problem 1753: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{5/2} (c+dx)^{1/6}} dx$$

Optimal (type 4, 858 leaves, 6 steps):

$$\begin{aligned}
 & - \frac{2 (c+dx)^{5/6}}{3 (bc-ad) (a+bx)^{3/2}} + \frac{8d (c+dx)^{5/6}}{9 (bc-ad)^2 \sqrt{a+bx}} + \\
 & \frac{8 (1+\sqrt{3}) d^2 \sqrt{a+bx} (c+dx)^{1/6}}{9 b^{2/3} (bc-ad)^2 \left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)} + \\
 & \left( 8d (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\
 & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] \right) / \\
 & \left( 3 \times 3^{3/4} b^{2/3} (bc-ad)^{5/3} \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right) + \\
 & \left( 4 (1-\sqrt{3}) d (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\
 & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] \right) / \\
 & \left( 9 \times 3^{1/4} b^{2/3} (bc-ad)^{5/3} \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right)
 \end{aligned}$$

Result (type 5, 105 leaves):

$$\begin{aligned}
 & - \left( \left( 2 (c+dx)^{5/6} \left( -5 (-3bc+7ad+4bdx) + 8d (a+bx) \sqrt{\frac{d(a+bx)}{-bc+ad}} \right. \right. \right. \\
 & \left. \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad} \right] \right) \right) / \left( 45 (bc-ad)^2 (a+bx)^{3/2} \right) \right)
 \end{aligned}$$



**Problem 1754: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{5/6}} dx$$

Optimal (type 4, 440 leaves, 5 steps):

$$\frac{81 (bc-ad)^2 \sqrt{a+bx} (c+dx)^{1/6}}{64 d^3} - \frac{9 (bc-ad) (a+bx)^{3/2} (c+dx)^{1/6}}{16 d^2} +$$

$$\frac{3 (a+bx)^{5/2} (c+dx)^{1/6}}{8 d} - \left( 81 \times 3^{3/4} (bc-ad)^{8/3} (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right.$$

$$\sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}}$$

$$\left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] \right) /$$

$$\left( 128 d^4 \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right)$$

Result (type 5, 138 leaves):

$$\frac{1}{64 d^4 \sqrt{a+bx}}$$

$$3 (c+dx)^{1/6} \left( d (a+bx) (47 a^2 d^2 + 2 a b d (-33 c + 14 d x) + b^2 (27 c^2 - 12 c d x + 8 d^2 x^2)) - \right.$$

$$\left. 81 (bc-ad)^3 \sqrt{\frac{d (a+bx)}{-bc+ad}} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c+dx)}{bc-ad} \right] \right)$$

**Problem 1755: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{5/6}} dx$$

Optimal (type 4, 405 leaves, 4 steps):

$$\begin{aligned}
 & - \frac{27 (bc - ad) \sqrt{a + bx} (c + dx)^{1/6}}{20 d^2} + \frac{3 (a + bx)^{3/2} (c + dx)^{1/6}}{5 d} + \\
 & \left( 27 \times 3^{3/4} (bc - ad)^{5/3} (c + dx)^{1/6} \left( (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right) \right. \\
 & \sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c + dx)^{1/3} + b^{2/3} (c + dx)^{2/3}}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3} \right)^2}} \\
 & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc - ad)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + dx)^{1/3}}{(bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3}} \right], \frac{1}{4} (2 + \sqrt{3}) \right] \right) / \\
 & \left( 40 d^3 \sqrt{a + bx} \sqrt{-\frac{b^{1/3} (c + dx)^{1/3} \left( (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right)}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3} \right)^2}} \right)
 \end{aligned}$$

Result (type 5, 107 leaves):

$$\begin{aligned}
 & \frac{1}{20 d^3 \sqrt{a + bx}} 3 (c + dx)^{1/6} \left( d (a + bx) (-9 bc + 13 ad + 4 b dx) + \right. \\
 & \left. 27 (bc - ad)^2 \sqrt{\frac{d (a + bx)}{-bc + ad}} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b (c + dx)}{bc - ad} \right] \right)
 \end{aligned}$$

**Problem 1756: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a + bx}}{(c + dx)^{5/6}} dx$$

Optimal (type 4, 372 leaves, 3 steps):

$$\frac{3 \sqrt{a+bx} (c+dx)^{1/6}}{2d} - \left( 3 \times 3^{3/4} (bc-ad)^{2/3} (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\ \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\ \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] \right) / \\ \left( 4d^2 \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right)$$

Result (type 5, 77 leaves):

$$\frac{3 \sqrt{a+bx} (c+dx)^{1/6} \left( 1 + \frac{{}_3\text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad} \right]}{\sqrt{\frac{d(a+bx)}{-bc+ad}}} \right)}{2d}$$

**Problem 1757: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{a+bx} (c+dx)^{5/6}} dx$$

Optimal (type 4, 343 leaves, 2 steps):

$$\left( 3^{3/4} (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\ \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\ \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] \right) / \\ \left( d (bc-ad)^{1/3} \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right)$$

Result (type 5, 71 leaves):

$$\frac{6 \sqrt{\frac{d(a+bx)}{-bc+ad}} (c+dx)^{1/6} \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad}\right]}{d \sqrt{a+bx}}$$

Problem 1758: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+bx)^{3/2} (c+dx)^{5/6}} dx$$

Optimal (type 4, 372 leaves, 3 steps):

$$\begin{aligned} & -\frac{2(c+dx)^{1/6}}{(bc-ad)\sqrt{a+bx}} - \left( 2(c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right) \right. \\ & \quad \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3} \right)^2}} \right. \\ & \quad \left. \text{EllipticF}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right) / \\ & \quad \left( 3^{1/4}(bc-ad)^{4/3} \sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3} \right)^2}} \right) \end{aligned}$$

Result (type 5, 82 leaves):

$$\frac{2(c+dx)^{1/6} \left( 1 + 2 \sqrt{\frac{d(a+bx)}{-bc+ad}} \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad}\right] \right)}{(bc-ad)\sqrt{a+bx}}$$

Problem 1759: Result unnecessarily involves higher level functions.

$$\int \frac{1}{(a+bx)^{5/2} (c+dx)^{5/6}} dx$$

Optimal (type 4, 410 leaves, 4 steps):

$$\begin{aligned}
 & -\frac{2(c+dx)^{1/6}}{3(bc-ad)(a+bx)^{3/2}} + \frac{16d(c+dx)^{1/6}}{9(bc-ad)^2\sqrt{a+bx}} + \\
 & \left( \frac{16d(c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right)}{\sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3} \right)^2}}} \right. \\
 & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}} \right], \frac{1}{4}(2+\sqrt{3}) \right] \right) / \\
 & \left( 9 \times 3^{1/4} (bc-ad)^{7/3} \sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3} \right)^2}} \right)
 \end{aligned}$$

Result (type 5, 102 leaves):

$$\begin{aligned}
 & \left( 2(c+dx)^{1/6} \left( -3bc + 11ad + 8bdx + 16d(a+bx) \sqrt{\frac{d(a+bx)}{-bc+ad}} \right. \right. \\
 & \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad} \right] \right) \right) / \left( 9(bc-ad)^2(a+bx)^{3/2} \right)
 \end{aligned}$$

**Problem 1760: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{5/2}}{(c+dx)^{7/6}} dx$$

Optimal (type 4, 880 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{6(a+bx)^{5/2}}{d(c+dx)^{1/6}} - \frac{405b(bc-ad)\sqrt{a+bx}(c+dx)^{5/6}}{56d^3} + \\
 & \frac{45b(a+bx)^{3/2}(c+dx)^{5/6}}{7d^2} - \frac{1215(1+\sqrt{3})b^{1/3}(bc-ad)^2\sqrt{a+bx}(c+dx)^{1/6}}{112d^3((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3})} - \\
 & \left( 1215 \times 3^{1/4} b^{1/3} (bc-ad)^{7/3} (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\
 & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\
 & \left. \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] \right) / \\
 & \left( 112 d^4 \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right) - \\
 & \left( 405 \times 3^{3/4} (1-\sqrt{3}) b^{1/3} (bc-ad)^{7/3} (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\
 & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\
 & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}} \right], \frac{1}{4} (2+\sqrt{3}) \right] \right) / \\
 & \left( 224 d^4 \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right)
 \end{aligned}$$

Result(type 5, 132 leaves):

$$\begin{aligned}
 & \frac{1}{56d^4\sqrt{a+bx}} 3(c+dx)^{5/6} \left( d(a+bx) \left( b(-23bc+31ad) + 8b^2dx - \frac{112(bc-ad)^2}{c+dx} \right) + \right. \\
 & \left. 81b(bc-ad)^2 \sqrt{\frac{d(a+bx)}{-bc+ad}} \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad} \right] \right)
 \end{aligned}$$

**Problem 1761: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{3/2}}{(c+dx)^{7/6}} dx$$

Optimal (type 4, 844 leaves, 6 steps):

$$\begin{aligned} & -\frac{6(a+bx)^{3/2}}{d(c+dx)^{1/6}} + \frac{27b\sqrt{a+bx}(c+dx)^{5/6}}{4d^2} + \frac{81(1+\sqrt{3})b^{1/3}(bc-ad)\sqrt{a+bx}(c+dx)^{1/6}}{8d^2\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)} + \\ & \left( \frac{81 \times 3^{1/4} b^{1/3} (bc-ad)^{4/3} (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}}} \right. \\ & \left. \text{EllipticE}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right) / \\ & \left( 8d^3 \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right) + \\ & \left( 27 \times 3^{3/4} (1-\sqrt{3}) b^{1/3} (bc-ad)^{4/3} (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\ & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/3} + b^{2/3} (c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right. \\ & \left. \text{EllipticF}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3}) b^{1/3} (c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right) / \\ & \left( 16d^3 \sqrt{a+bx} \sqrt{-\frac{b^{1/3} (c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3}) b^{1/3} (c+dx)^{1/3} \right)^2}} \right) \end{aligned}$$

Result (type 5, 99 leaves):

$$\frac{1}{20d^2} 3\sqrt{a+bx}(c+dx)^{5/6} \left( \frac{5(9bc-8ad+bdx)}{c+dx} + \frac{27b \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad}\right]}{\sqrt{\frac{d(a+bx)}{-bc+ad}}} \right)$$

**Problem 1762: Result unnecessarily involves higher level functions.**

$$\int \frac{\sqrt{a+bx}}{(c+dx)^{7/6}} dx$$

Optimal (type 4, 806 leaves, 5 steps):

$$\frac{6\sqrt{a+bx}}{d(c+dx)^{1/6}} - \frac{9(1+\sqrt{3})b^{1/3}\sqrt{a+bx}(c+dx)^{1/6}}{d((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3})} - \left( \frac{9 \times 3^{1/4} b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/6} ((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3})}{\sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3})^2}}}} \right. \\ \left. \text{EllipticE}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right]\right) / \\ \left( d^2\sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3}((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3})}{((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3})^2}} \right) - \\ \left( 3 \times 3^{3/4} (1-\sqrt{3}) b^{1/3} (bc-ad)^{1/3} (c+dx)^{1/6} ((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}) \right. \\ \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3})^2}} \right) \\ \left. \text{EllipticF}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right]\right) / \\ \left( 2d^2\sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3}((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3})}{((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3})^2}} \right)$$

Result (type 5, 90 leaves):

$$\left( -30d(a+bx) + 18b \sqrt{\frac{d(a+bx)}{-bc+ad}} (c+dx) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad}\right] \right) / \\ (5d^2\sqrt{a+bx}(c+dx)^{1/6})$$



**Problem 1763: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{\sqrt{a+bx} (c+dx)^{7/6}} dx$$

Optimal (type 4, 817 leaves, 5 steps):

$$\begin{aligned} & \frac{6\sqrt{a+bx}}{(bc-ad)(c+dx)^{1/6}} + \frac{6(1+\sqrt{3})b^{1/3}\sqrt{a+bx}(c+dx)^{1/6}}{(bc-ad)\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)} + \\ & \left( 6 \times 3^{1/4} b^{1/3} (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\ & \quad \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}} \\ & \quad \left. \text{EllipticE}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right) / \\ & \left( d(bc-ad)^{2/3}\sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}} \right) + \\ & \left( 3^{3/4}(1-\sqrt{3})b^{1/3}(c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right) \right. \\ & \quad \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}} \\ & \quad \left. \text{EllipticF}\left[\text{ArcCos}\left[\frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}}\right], \frac{1}{4}(2+\sqrt{3})\right] \right) / \\ & \left( d(bc-ad)^{2/3}\sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3}\left((bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3}\right)}{\left((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}\right)^2}} \right) \end{aligned}$$

Result (type 5, 100 leaves):

$$\begin{aligned} & \left( 6 \left( 5d(a+bx) - 2b\sqrt{\frac{d(a+bx)}{-bc+ad}}(c+dx) \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) / \\ & (5d(bc-ad)\sqrt{a+bx}(c+dx)^{1/6}) \end{aligned}$$

**Problem 1764: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{3/2} (c+dx)^{7/6}} dx$$

Optimal (type 4, 844 leaves, 6 steps):

$$\begin{aligned} & - \frac{2}{(bc-ad)\sqrt{a+bx}(c+dx)^{1/6}} - \frac{8d\sqrt{a+bx}}{(bc-ad)^2(c+dx)^{1/6}} - \\ & \frac{8(1+\sqrt{3})b^{1/3}d\sqrt{a+bx}(c+dx)^{1/6}}{(bc-ad)^2((bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3})} - \\ & \left( 8 \times 3^{1/4} b^{1/3} (c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3} (c+dx)^{1/3} \right) \right. \\ & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3} \right)^2}} \right. \\ & \left. \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}} \right], \frac{1}{4}(2+\sqrt{3}) \right] \right) / \\ & \left( (bc-ad)^{5/3} \sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3} \right)^2}} \right) - \\ & \left( 4(1-\sqrt{3})b^{1/3}(c+dx)^{1/6} \left( (bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right) \right. \\ & \left. \sqrt{\frac{(bc-ad)^{2/3} + b^{1/3}(bc-ad)^{1/3}(c+dx)^{1/3} + b^{2/3}(c+dx)^{2/3}}{\left( (bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3} \right)^2}} \right. \\ & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc-ad)^{1/3} - (1-\sqrt{3})b^{1/3}(c+dx)^{1/3}}{(bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3}} \right], \frac{1}{4}(2+\sqrt{3}) \right] \right) / \\ & \left( 3^{1/4} (bc-ad)^{5/3} \sqrt{a+bx} \sqrt{-\frac{b^{1/3}(c+dx)^{1/3} \left( (bc-ad)^{1/3} - b^{1/3}(c+dx)^{1/3} \right)}{\left( (bc-ad)^{1/3} - (1+\sqrt{3})b^{1/3}(c+dx)^{1/3} \right)^2}} \right) \end{aligned}$$

Result (type 5, 102 leaves):

$$\left. \left( \left( 2 \left( 15 a d + 5 b (c + 4 d x) - 8 b \sqrt{\frac{d (a + b x)}{-b c + a d}} (c + d x) \right. \right. \right. \right. \\
 \left. \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d} \right] \right) \right) \right) / \left( 5 (b c - a d)^2 \sqrt{a + b x} (c + d x)^{1/6} \right)$$

**Problem 1765: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a + b x)^{5/2} (c + d x)^{7/6}} dx$$

Optimal (type 4, 893 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{2}{3 (bc - ad) (a + bx)^{3/2} (c + dx)^{1/6}} + \frac{20d}{9 (bc - ad)^2 \sqrt{a + bx} (c + dx)^{1/6}} + \\
 & \frac{80d^2 \sqrt{a + bx}}{9 (bc - ad)^3 (c + dx)^{1/6}} + \frac{80 (1 + \sqrt{3}) b^{1/3} d^2 \sqrt{a + bx} (c + dx)^{1/6}}{9 (bc - ad)^3 \left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3} \right)} + \\
 & \left( \frac{80 b^{1/3} d (c + dx)^{1/6} \left( (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right)}{\sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c + dx)^{1/3} + b^{2/3} (c + dx)^{2/3}}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3} \right)^2}}} \right. \\
 & \left. \text{EllipticE} \left[ \text{ArcCos} \left[ \frac{(bc - ad)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + dx)^{1/3}}{(bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3}}, \frac{1}{4} (2 + \sqrt{3}) \right] \right] \right) / \\
 & \left( 3 \times 3^{3/4} (bc - ad)^{8/3} \sqrt{a + bx} \sqrt{-\frac{b^{1/3} (c + dx)^{1/3} \left( (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right)}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3} \right)^2}} \right) + \\
 & \left( \frac{40 (1 - \sqrt{3}) b^{1/3} d (c + dx)^{1/6} \left( (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right)}{\sqrt{\frac{(bc - ad)^{2/3} + b^{1/3} (bc - ad)^{1/3} (c + dx)^{1/3} + b^{2/3} (c + dx)^{2/3}}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3} \right)^2}}} \right. \\
 & \left. \text{EllipticF} \left[ \text{ArcCos} \left[ \frac{(bc - ad)^{1/3} - (1 - \sqrt{3}) b^{1/3} (c + dx)^{1/3}}{(bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3}}, \frac{1}{4} (2 + \sqrt{3}) \right] \right] \right) / \\
 & \left( 9 \times 3^{1/4} (bc - ad)^{8/3} \sqrt{a + bx} \sqrt{-\frac{b^{1/3} (c + dx)^{1/3} \left( (bc - ad)^{1/3} - b^{1/3} (c + dx)^{1/3} \right)}{\left( (bc - ad)^{1/3} - (1 + \sqrt{3}) b^{1/3} (c + dx)^{1/3} \right)^2}} \right)
 \end{aligned}$$

Result (type 5, 139 leaves):

$$\begin{aligned}
 & - \left( \left( 2 \left( 27 a^2 d^2 + 2 a b d (8 c + 35 d x) + b^2 (-3 c^2 + 10 c d x + 40 d^2 x^2) - \right. \right. \right. \\
 & \quad \left. \left. \left. 16 b d (a + b x) \sqrt{\frac{d (a + b x)}{-b c + a d}} (c + d x) \operatorname{Hypergeometric2F1} \left[ \frac{1}{2}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d} \right] \right) \right) / \\
 & \quad \left. \left( 9 (-b c + a d)^3 (a + b x)^{3/2} (c + d x)^{1/6} \right) \right)
 \end{aligned}$$

**Problem 1766: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{1/6} (c + d x)^{13/6} dx$$

Optimal (type 5, 84 leaves, 2 steps):

$$\begin{aligned}
 & \frac{1}{7 b^3 \left( \frac{b (c + d x)}{b c - a d} \right)^{1/6}} \\
 & 6 (b c - a d)^2 (a + b x)^{7/6} (c + d x)^{1/6} \operatorname{Hypergeometric2F1} \left[ -\frac{13}{6}, \frac{7}{6}, \frac{13}{6}, -\frac{d (a + b x)}{b c - a d} \right]
 \end{aligned}$$

Result (type 5, 182 leaves):

$$\begin{aligned}
 & - \left( \left( 3 (c + d x)^{1/6} \left( -d (a + b x) (91 a^3 d^3 - 13 a^2 b d^2 (23 c + 2 d x) + a b^2 d (341 c^2 + 84 c d x + 16 d^2 x^2) + \right. \right. \right. \\
 & \quad \left. \left. \left. b^3 (91 c^3 + 614 c^2 d x + 656 c d^2 x^2 + 224 d^3 x^3) \right) + 91 (b c - a d)^4 \left( \frac{d (a + b x)}{-b c + a d} \right)^{5/6} \right. \right. \\
 & \quad \left. \left. \operatorname{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{5}{6}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d} \right] \right) \right) / \left( 2240 b^3 d^2 (a + b x)^{5/6} \right)
 \end{aligned}$$

**Problem 1772: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^{1/6} (c + d x)^{5/6} dx$$

Optimal (type 3, 427 leaves, 14 steps):

$$\frac{5 (b c - a d) (a + b x)^{1/6} (c + d x)^{5/6}}{12 b d} + \frac{(a + b x)^{7/6} (c + d x)^{5/6}}{2 b} +$$

$$\frac{5 (b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a + b x)^{1/6}}{\sqrt{3} b^{1/6} (c + d x)^{1/6}}\right]}{24 \sqrt{3} b^{11/6} d^{7/6}} - \frac{5 (b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a + b x)^{1/6}}{\sqrt{3} b^{1/6} (c + d x)^{1/6}}\right]}{24 \sqrt{3} b^{11/6} d^{7/6}} -$$

$$\frac{5 (b c - a d)^2 \operatorname{ArcTanh}\left[\frac{d^{1/6} (a + b x)^{1/6}}{b^{1/6} (c + d x)^{1/6}}\right]}{36 b^{11/6} d^{7/6}} + \frac{5 (b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a + b x)^{1/3}}{(c + d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a + b x)^{1/6}}{(c + d x)^{1/6}}\right]}{144 b^{11/6} d^{7/6}} -$$

$$\frac{5 (b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a + b x)^{1/3}}{(c + d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a + b x)^{1/6}}{(c + d x)^{1/6}}\right]}{144 b^{11/6} d^{7/6}}$$

Result (type 5, 109 leaves):

$$\left( (c + d x)^{5/6} \left( d (a + b x) (5 b c + a d + 6 b d x) - (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{5/6} \right. \right.$$

$$\left. \left. \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d}\right] \right) \right) / (12 b d^2 (a + b x)^{5/6})$$

### Problem 1773: Result unnecessarily involves higher level functions.

$$\int \frac{(a + b x)^{1/6}}{(c + d x)^{1/6}} dx$$

Optimal (type 3, 378 leaves, 13 steps):

$$\frac{(a + b x)^{1/6} (c + d x)^{5/6}}{d} + \frac{(b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a + b x)^{1/6}}{\sqrt{3} b^{1/6} (c + d x)^{1/6}}\right]}{2 \sqrt{3} b^{5/6} d^{7/6}} -$$

$$\frac{(b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a + b x)^{1/6}}{\sqrt{3} b^{1/6} (c + d x)^{1/6}}\right]}{2 \sqrt{3} b^{5/6} d^{7/6}} - \frac{(b c - a d) \operatorname{ArcTanh}\left[\frac{d^{1/6} (a + b x)^{1/6}}{b^{1/6} (c + d x)^{1/6}}\right]}{3 b^{5/6} d^{7/6}} +$$

$$\frac{(b c - a d) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a + b x)^{1/3}}{(c + d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a + b x)^{1/6}}{(c + d x)^{1/6}}\right]}{12 b^{5/6} d^{7/6}} -$$

$$\frac{(b c - a d) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a + b x)^{1/3}}{(c + d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a + b x)^{1/6}}{(c + d x)^{1/6}}\right]}{12 b^{5/6} d^{7/6}}$$

Result (type 5, 76 leaves):

$$\frac{(a + b x)^{1/6} (c + d x)^{5/6} \left( 5 + \frac{\operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b (c + d x)}{b c - a d}\right]}{\left(\frac{d (a + b x)}{-b c + a d}\right)^{1/6}} \right)}{5 d}$$

**Problem 1774: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{1/6}}{(c+dx)^{7/6}} dx$$

Optimal (type 3, 332 leaves, 13 steps):

$$\begin{aligned} & -\frac{6(a+bx)^{1/6}}{d(c+dx)^{1/6}} - \frac{\sqrt{3} b^{1/6} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3} b^{1/6}(c+dx)^{1/6}}\right]}{d^{7/6}} + \\ & \frac{\sqrt{3} b^{1/6} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3} b^{1/6}(c+dx)^{1/6}}\right]}{d^{7/6}} + \frac{2 b^{1/6} \operatorname{ArcTanh}\left[\frac{d^{1/6}(a+bx)^{1/6}}{b^{1/6}(c+dx)^{1/6}}\right]}{d^{7/6}} - \\ & \frac{b^{1/6} \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6} d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{2 d^{7/6}} + \frac{b^{1/6} \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6} d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{2 d^{7/6}} \end{aligned}$$

Result (type 5, 89 leaves):

$$\left( 6 \left( -5d(a+bx) + b \left( \frac{d(a+bx)}{-bc+ad} \right)^{5/6} (c+dx) \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) / \left( 5d^2(a+bx)^{5/6}(c+dx)^{1/6} \right)$$

**Problem 1779: Result unnecessarily involves higher level functions.**

$$\int (a+bx)^{5/6} (c+dx)^{1/6} dx$$

Optimal (type 3, 427 leaves, 14 steps):

$$\begin{aligned} & \frac{(bc-ad)(a+bx)^{5/6}(c+dx)^{1/6}}{12bd} + \frac{(a+bx)^{11/6}(c+dx)^{1/6}}{2b} - \\ & \frac{5(bc-ad)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3} b^{1/6}(c+dx)^{1/6}}\right]}{24\sqrt{3} b^{7/6} d^{11/6}} + \frac{5(bc-ad)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3} b^{1/6}(c+dx)^{1/6}}\right]}{24\sqrt{3} b^{7/6} d^{11/6}} - \\ & \frac{5(bc-ad)^2 \operatorname{ArcTanh}\left[\frac{d^{1/6}(a+bx)^{1/6}}{b^{1/6}(c+dx)^{1/6}}\right]}{36 b^{7/6} d^{11/6}} + \frac{5(bc-ad)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6} d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{144 b^{7/6} d^{11/6}} - \\ & \frac{5(bc-ad)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6} d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{144 b^{7/6} d^{11/6}} \end{aligned}$$

Result (type 5, 109 leaves):

$$\left( (c+dx)^{1/6} \left( d(a+bx)(5ad+b(c+6dx)) - 5(bc-ad)^2 \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/6} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad}\right] \right) \right) / \left( 12bd^2(a+bx)^{1/6} \right)$$

**Problem 1780: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{5/6}} dx$$

Optimal (type 3, 378 leaves, 13 steps):

$$\frac{(a+bx)^{5/6} (c+dx)^{1/6}}{d} - \frac{5 (bc-ad) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}}\right]}{2\sqrt{3}b^{1/6}d^{11/6}} +$$

$$\frac{5 (bc-ad) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}}\right]}{2\sqrt{3}b^{1/6}d^{11/6}} - \frac{5 (bc-ad) \operatorname{ArcTanh}\left[\frac{d^{1/6}(a+bx)^{1/6}}{b^{1/6}(c+dx)^{1/6}}\right]}{3b^{1/6}d^{11/6}} +$$

$$\frac{5 (bc-ad) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{12b^{1/6}d^{11/6}} -$$

$$\frac{5 (bc-ad) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{12b^{1/6}d^{11/6}}$$

Result (type 5, 74 leaves):

$$\frac{(a+bx)^{5/6} (c+dx)^{1/6}}{d} \left( 1 + \frac{5 \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad}\right]}{\left(\frac{d(a+bx)}{-bc+ad}\right)^{5/6}} \right)$$

**Problem 1781: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{5/6}}{(c+dx)^{11/6}} dx$$

Optimal (type 3, 334 leaves, 13 steps):

$$-\frac{6(a+bx)^{5/6}}{5d(c+dx)^{5/6}} + \frac{\sqrt{3}b^{5/6} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}}\right]}{d^{11/6}} -$$

$$\frac{\sqrt{3}b^{5/6} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}}\right]}{d^{11/6}} + \frac{2b^{5/6} \operatorname{ArcTanh}\left[\frac{d^{1/6}(a+bx)^{1/6}}{b^{1/6}(c+dx)^{1/6}}\right]}{d^{11/6}} -$$

$$\frac{b^{5/6} \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{2d^{11/6}} + \frac{b^{5/6} \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{2d^{11/6}}$$

Result (type 5, 90 leaves):

$$\left( -6d(a+bx) + 30b \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/6} (c+dx) \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad}\right] \right) /$$

$$\left( 5d^2(a+bx)^{1/6} (c+dx)^{5/6} \right)$$



**Problem 1792: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{7/6} (c + d x)^{13/6} dx$$

Optimal (type 5, 84 leaves, 2 steps):

$$\frac{1}{13 b^3 \left( \frac{b(c+dx)}{bc-ad} \right)^{1/6}}$$

$$6 (bc - ad)^2 (a + b x)^{13/6} (c + d x)^{1/6} \text{Hypergeometric2F1} \left[ -\frac{13}{6}, \frac{13}{6}, \frac{19}{6}, -\frac{d(a+bx)}{bc-ad} \right]$$

Result (type 5, 234 leaves):

$$\begin{aligned} & -\frac{1}{8320 b^3 d^3 (a + b x)^{5/6}} \\ & 3 (c + d x)^{1/6} \left( -d (a + b x) (91 a^4 d^4 - 26 a^3 b d^3 (15 c + d x) + 2 a^2 b^2 d^2 (320 c^2 + 55 c d x + 8 d^2 x^2) + \right. \\ & \quad \left. 2 a b^3 d (195 c^3 + 1225 c^2 d x + 1280 c d^2 x^2 + 432 d^3 x^3) + \right. \\ & \quad \left. b^4 (-91 c^4 + 26 c^3 d x + 1264 c^2 d^2 x^2 + 1696 c d^3 x^3 + 640 d^4 x^4) \right) - \\ & 91 (bc - ad)^5 \left( \frac{d(a+bx)}{-bc+ad} \right)^{5/6} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{5}{6}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad} \right] \end{aligned}$$

**Problem 1793: Result more than twice size of optimal antiderivative.**

$$\int (a + b x)^{7/6} (c + d x)^{7/6} dx$$

Optimal (type 5, 82 leaves, 2 steps):

$$\frac{1}{13 b^2 \left( \frac{b(c+dx)}{bc-ad} \right)^{1/6}}$$

$$6 (bc - ad) (a + b x)^{13/6} (c + d x)^{1/6} \text{Hypergeometric2F1} \left[ -\frac{7}{6}, \frac{13}{6}, \frac{19}{6}, -\frac{d(a+bx)}{bc-ad} \right]$$

Result (type 5, 183 leaves):

$$\begin{aligned} & \left( 3 (c + d x)^{1/6} \left( -d (a + b x) (7 a^3 d^3 - a^2 b d^2 (23 c + 2 d x) - a b^2 d (23 c^2 + 92 c d x + 48 d^2 x^2) + \right. \right. \\ & \quad \left. \left. b^3 (7 c^3 - 2 c^2 d x - 48 c d^2 x^2 - 32 d^3 x^3) \right) + 7 (bc - ad)^4 \left( \frac{d(a+bx)}{-bc+ad} \right)^{5/6} \right. \\ & \quad \left. \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{5}{6}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad} \right] \right) / (320 b^2 d^3 (a + b x)^{5/6}) \end{aligned}$$

**Problem 1798: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{1/6}} dx$$

Optimal (type 3, 424 leaves, 14 steps):

$$\begin{aligned} & - \frac{7 (bc - ad) (a+bx)^{1/6} (c+dx)^{5/6}}{12 d^2} + \frac{(a+bx)^{7/6} (c+dx)^{5/6}}{2 d} \\ & - \frac{7 (bc - ad)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+bx)^{1/6}}{\sqrt{3} b^{1/6} (c+dx)^{1/6}}\right]}{24 \sqrt{3} b^{5/6} d^{13/6}} + \frac{7 (bc - ad)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+bx)^{1/6}}{\sqrt{3} b^{1/6} (c+dx)^{1/6}}\right]}{24 \sqrt{3} b^{5/6} d^{13/6}} + \\ & - \frac{7 (bc - ad)^2 \operatorname{ArcTanh}\left[\frac{d^{1/6} (a+bx)^{1/6}}{b^{1/6} (c+dx)^{1/6}}\right]}{36 b^{5/6} d^{13/6}} - \frac{7 (bc - ad)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{144 b^{5/6} d^{13/6}} + \\ & - \frac{7 (bc - ad)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{144 b^{5/6} d^{13/6}} \end{aligned}$$

Result (type 5, 108 leaves):

$$\begin{aligned} & \frac{1}{60 d^3 (a+bx)^{5/6}} (c+dx)^{5/6} \left( 5 d (a+bx) (-7bc + 13ad + 6bdx) + \right. \\ & \left. 7 (bc - ad)^2 \left( \frac{d (a+bx)}{-bc + ad} \right)^{5/6} \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b (c+dx)}{bc - ad}\right] \right) \end{aligned}$$

**Problem 1799: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+bx)^{7/6}}{(c+dx)^{7/6}} dx$$

Optimal (type 3, 403 leaves, 14 steps):

$$\begin{aligned} & - \frac{6 (a+bx)^{7/6}}{d (c+dx)^{1/6}} + \frac{7 b (a+bx)^{1/6} (c+dx)^{5/6}}{d^2} + \frac{7 b^{1/6} (bc - ad) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+bx)^{1/6}}{\sqrt{3} b^{1/6} (c+dx)^{1/6}}\right]}{2 \sqrt{3} d^{13/6}} - \\ & - \frac{7 b^{1/6} (bc - ad) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+bx)^{1/6}}{\sqrt{3} b^{1/6} (c+dx)^{1/6}}\right]}{2 \sqrt{3} d^{13/6}} - \frac{7 b^{1/6} (bc - ad) \operatorname{ArcTanh}\left[\frac{d^{1/6} (a+bx)^{1/6}}{b^{1/6} (c+dx)^{1/6}}\right]}{3 d^{13/6}} + \\ & - \frac{7 b^{1/6} (bc - ad) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{12 d^{13/6}} - \\ & - \frac{7 b^{1/6} (bc - ad) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{12 d^{13/6}} \end{aligned}$$

Result (type 5, 99 leaves):

$$\frac{1}{5 d^2} (a + b x)^{1/6} (c + d x)^{5/6} \left( \frac{5 (7 b c - 6 a d + b d x)}{c + d x} + \frac{7 b \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad}\right]}{\left(\frac{d(a+bx)}{-bc+ad}\right)^{1/6}} \right)$$

**Problem 1800: Result unnecessarily involves higher level functions.**

$$\int \frac{(a + b x)^{7/6}}{(c + d x)^{13/6}} dx$$

Optimal (type 3, 358 leaves, 14 steps):

$$\begin{aligned} & -\frac{6(a+bx)^{7/6}}{7d(c+dx)^{7/6}} - \frac{6b(a+bx)^{1/6}}{d^2(c+dx)^{1/6}} - \frac{\sqrt{3} b^{7/6} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3} b^{1/6}(c+dx)^{1/6}}\right]}{d^{13/6}} + \\ & \frac{\sqrt{3} b^{7/6} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3} b^{1/6}(c+dx)^{1/6}}\right]}{d^{13/6}} + \frac{2 b^{7/6} \operatorname{ArcTanh}\left[\frac{d^{1/6}(a+bx)^{1/6}}{b^{1/6}(c+dx)^{1/6}}\right]}{d^{13/6}} - \\ & \frac{b^{7/6} \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6} d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{2 d^{13/6}} + \frac{b^{7/6} \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6} d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{2 d^{13/6}} \end{aligned}$$

Result (type 5, 107 leaves):

$$\begin{aligned} & \left( -30 d (a + b x) (7 b c + a d + 8 b d x) + 42 b^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{5/6} (c + d x)^2 \right. \\ & \left. \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad}\right] \right) / \left( 35 d^3 (a + b x)^{5/6} (c + d x)^{7/6} \right) \end{aligned}$$

**Problem 1805: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + d x)^{7/6}}{(a + b x)^{1/6}} dx$$

Optimal (type 3, 424 leaves, 14 steps):

$$\begin{aligned} & \frac{7(b c - a d)(a + b x)^{5/6}(c + d x)^{1/6}}{12 b^2} + \frac{(a + b x)^{5/6}(c + d x)^{7/6}}{2 b} + \\ & \frac{7(b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3} b^{1/6}(c+dx)^{1/6}}\right]}{24 \sqrt{3} b^{13/6} d^{5/6}} - \frac{7(b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3} b^{1/6}(c+dx)^{1/6}}\right]}{24 \sqrt{3} b^{13/6} d^{5/6}} + \\ & \frac{7(b c - a d)^2 \operatorname{ArcTanh}\left[\frac{d^{1/6}(a+bx)^{1/6}}{b^{1/6}(c+dx)^{1/6}}\right]}{36 b^{13/6} d^{5/6}} - \frac{7(b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6} d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{144 b^{13/6} d^{5/6}} + \\ & \frac{7(b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6} d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{144 b^{13/6} d^{5/6}} \end{aligned}$$

Result (type 5, 111 leaves):

$$\left( (c+dx)^{1/6} \left( -d(a+bx) (-13bc+7ad-6bdx) + 7(bc-ad)^2 \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/6} \right. \right. \\ \left. \left. \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad} \right] \right) \right) / (12b^2d(a+bx)^{1/6})$$

**Problem 1806: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{1/6}}{(a+bx)^{1/6}} dx$$

Optimal (type 3, 378 leaves, 13 steps):

$$\frac{(a+bx)^{5/6} (c+dx)^{1/6}}{b} + \frac{(bc-ad) \text{ArcTan} \left[ \frac{1}{\sqrt{3}} - \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}} \right]}{2\sqrt{3}b^{7/6}d^{5/6}} - \\ \frac{(bc-ad) \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}} \right]}{2\sqrt{3}b^{7/6}d^{5/6}} + \frac{(bc-ad) \text{ArcTanh} \left[ \frac{d^{1/6}(a+bx)^{1/6}}{b^{1/6}(c+dx)^{1/6}} \right]}{3b^{7/6}d^{5/6}} - \\ \frac{(bc-ad) \text{Log} \left[ b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}} \right]}{12b^{7/6}d^{5/6}} + \\ \frac{(bc-ad) \text{Log} \left[ b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}} \right]}{12b^{7/6}d^{5/6}}$$

Result (type 5, 90 leaves):

$$\frac{1}{bd(a+bx)^{1/6}} \\ (c+dx)^{1/6} \left( d(a+bx) + (bc-ad) \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/6} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad} \right] \right)$$

**Problem 1807: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{1/6} (c+dx)^{5/6}} dx$$

Optimal (type 3, 309 leaves, 12 steps):

$$\frac{\sqrt{3} \text{ArcTan} \left[ \frac{1}{\sqrt{3}} - \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}} \right]}{b^{1/6}d^{5/6}} - \frac{\sqrt{3} \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}} \right]}{b^{1/6}d^{5/6}} + \frac{2 \text{ArcTanh} \left[ \frac{d^{1/6}(a+bx)^{1/6}}{b^{1/6}(c+dx)^{1/6}} \right]}{b^{1/6}d^{5/6}} - \\ \frac{\text{Log} \left[ b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}} \right]}{2b^{1/6}d^{5/6}} + \frac{\text{Log} \left[ b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}} \right]}{2b^{1/6}d^{5/6}}$$

Result (type 5, 71 leaves):

$$\frac{6 \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/6} (c+dx)^{1/6} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad} \right]}{d (a+bx)^{1/6}}$$

**Problem 1824: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{11/6}}{(a+bx)^{5/6}} dx$$

Optimal (type 3, 424 leaves, 14 steps):

$$\begin{aligned} & \frac{11 (bc-ad) (a+bx)^{1/6} (c+dx)^{5/6}}{12 b^2} + \frac{(a+bx)^{1/6} (c+dx)^{11/6}}{2 b} - \\ & \frac{55 (bc-ad)^2 \text{ArcTan} \left[ \frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a+bx)^{1/6}}{\sqrt{3} b^{1/6} (c+dx)^{1/6}} \right]}{24 \sqrt{3} b^{17/6} d^{1/6}} + \frac{55 (bc-ad)^2 \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a+bx)^{1/6}}{\sqrt{3} b^{1/6} (c+dx)^{1/6}} \right]}{24 \sqrt{3} b^{17/6} d^{1/6}} + \\ & \frac{55 (bc-ad)^2 \text{ArcTanh} \left[ \frac{d^{1/6} (a+bx)^{1/6}}{b^{1/6} (c+dx)^{1/6}} \right]}{36 b^{17/6} d^{1/6}} - \frac{55 (bc-ad)^2 \text{Log} \left[ b^{1/3} + \frac{d^{1/3} (a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6} d^{1/6} (a+bx)^{1/6}}{(c+dx)^{1/6}} \right]}{144 b^{17/6} d^{1/6}} + \\ & \frac{55 (bc-ad)^2 \text{Log} \left[ b^{1/3} + \frac{d^{1/3} (a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6} d^{1/6} (a+bx)^{1/6}}{(c+dx)^{1/6}} \right]}{144 b^{17/6} d^{1/6}} \end{aligned}$$

Result (type 5, 111 leaves):

$$\left( (c+dx)^{5/6} \left( -d(a+bx) (-17bc+11ad-6bdx) + 11(bc-ad)^2 \left( \frac{d(a+bx)}{-bc+ad} \right)^{5/6} \right. \right. \\ \left. \left. \text{Hypergeometric2F1} \left[ \frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad} \right] \right) \right) / (12 b^2 d (a+bx)^{5/6})$$

**Problem 1825: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{5/6}}{(a+bx)^{5/6}} dx$$

Optimal (type 3, 378 leaves, 13 steps):

$$\frac{(a+bx)^{1/6} (c+dx)^{5/6}}{b} - \frac{5 (bc-ad) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}}\right]}{2\sqrt{3}b^{11/6}d^{1/6}} +$$

$$\frac{5 (bc-ad) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}}\right]}{2\sqrt{3}b^{11/6}d^{1/6}} + \frac{5 (bc-ad) \operatorname{ArcTanh}\left[\frac{d^{1/6}(a+bx)^{1/6}}{b^{1/6}(c+dx)^{1/6}}\right]}{3b^{11/6}d^{1/6}} -$$

$$\frac{5 (bc-ad) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{12b^{11/6}d^{1/6}} +$$

$$\frac{5 (bc-ad) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{12b^{11/6}d^{1/6}}$$

Result (type 5, 90 leaves):

$$\frac{1}{bd(a+bx)^{5/6}} (c+dx)^{5/6} \left( d(a+bx) + (bc-ad) \left( \frac{d(a+bx)}{-bc+ad} \right)^{5/6} \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad}\right] \right)$$

**Problem 1826: Result unnecessarily involves higher level functions.**

$$\int \frac{1}{(a+bx)^{5/6} (c+dx)^{1/6}} dx$$

Optimal (type 3, 309 leaves, 12 steps):

$$-\frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}}\right]}{b^{5/6}d^{1/6}} + \frac{\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3}b^{1/6}(c+dx)^{1/6}}\right]}{b^{5/6}d^{1/6}} + \frac{2 \operatorname{ArcTanh}\left[\frac{d^{1/6}(a+bx)^{1/6}}{b^{1/6}(c+dx)^{1/6}}\right]}{b^{5/6}d^{1/6}} -$$

$$\frac{\operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{2b^{5/6}d^{1/6}} + \frac{\operatorname{Log}\left[b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6}d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}}\right]}{2b^{5/6}d^{1/6}}$$

Result (type 5, 73 leaves):

$$\frac{6 \left( \frac{d(a+bx)}{-bc+ad} \right)^{5/6} (c+dx)^{5/6} \operatorname{Hypergeometric2F1}\left[\frac{5}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad}\right]}{5d(a+bx)^{5/6}}$$

**Problem 1831: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{13/6}}{(a+bx)^{7/6}} dx$$

Optimal (type 3, 449 leaves, 15 steps):

$$\begin{aligned}
 & \frac{91 d (b c - a d) (a + b x)^{5/6} (c + d x)^{1/6}}{12 b^3} + \frac{13 d (a + b x)^{5/6} (c + d x)^{7/6}}{2 b^2} - \\
 & \frac{6 (c + d x)^{13/6}}{b (a + b x)^{1/6}} + \frac{91 d^{1/6} (b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a + b x)^{1/6}}{\sqrt{3} b^{1/6} (c + d x)^{1/6}}\right]}{24 \sqrt{3} b^{19/6}} - \\
 & \frac{91 d^{1/6} (b c - a d)^2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a + b x)^{1/6}}{\sqrt{3} b^{1/6} (c + d x)^{1/6}}\right]}{24 \sqrt{3} b^{19/6}} + \frac{91 d^{1/6} (b c - a d)^2 \operatorname{ArcTanh}\left[\frac{d^{1/6} (a + b x)^{1/6}}{b^{1/6} (c + d x)^{1/6}}\right]}{36 b^{19/6}} - \\
 & \frac{91 d^{1/6} (b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a + b x)^{1/3}}{(c + d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a + b x)^{1/6}}{(c + d x)^{1/6}}\right]}{144 b^{19/6}} + \\
 & \frac{91 d^{1/6} (b c - a d)^2 \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a + b x)^{1/3}}{(c + d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a + b x)^{1/6}}{(c + d x)^{1/6}}\right]}{144 b^{19/6}}
 \end{aligned}$$

Result (type 5, 129 leaves):

$$\begin{aligned}
 & \frac{1}{12 b^3 (a + b x)^{1/6}} (c + d x)^{1/6} \left( -91 a^2 d^2 - 13 a b d (-13 c + d x) + b^2 (-72 c^2 + 25 c d x + 6 d^2 x^2) + \right. \\
 & \left. 91 (b c - a d)^2 \left( \frac{d (a + b x)}{-b c + a d} \right)^{1/6} \operatorname{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b (c + d x)}{b c - a d}\right] \right)
 \end{aligned}$$

**Problem 1832: Result unnecessarily involves higher level functions.**

$$\int \frac{(c + d x)^{7/6}}{(a + b x)^{7/6}} dx$$

Optimal (type 3, 403 leaves, 14 steps):

$$\begin{aligned}
 & \frac{7 d (a + b x)^{5/6} (c + d x)^{1/6}}{b^2} - \frac{6 (c + d x)^{7/6}}{b (a + b x)^{1/6}} + \frac{7 d^{1/6} (b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2 d^{1/6} (a + b x)^{1/6}}{\sqrt{3} b^{1/6} (c + d x)^{1/6}}\right]}{2 \sqrt{3} b^{13/6}} - \\
 & \frac{7 d^{1/6} (b c - a d) \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2 d^{1/6} (a + b x)^{1/6}}{\sqrt{3} b^{1/6} (c + d x)^{1/6}}\right]}{2 \sqrt{3} b^{13/6}} + \frac{7 d^{1/6} (b c - a d) \operatorname{ArcTanh}\left[\frac{d^{1/6} (a + b x)^{1/6}}{b^{1/6} (c + d x)^{1/6}}\right]}{3 b^{13/6}} - \\
 & \frac{7 d^{1/6} (b c - a d) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a + b x)^{1/3}}{(c + d x)^{1/3}} - \frac{b^{1/6} d^{1/6} (a + b x)^{1/6}}{(c + d x)^{1/6}}\right]}{12 b^{13/6}} + \\
 & \frac{7 d^{1/6} (b c - a d) \operatorname{Log}\left[b^{1/3} + \frac{d^{1/3} (a + b x)^{1/3}}{(c + d x)^{1/3}} + \frac{b^{1/6} d^{1/6} (a + b x)^{1/6}}{(c + d x)^{1/6}}\right]}{12 b^{13/6}}
 \end{aligned}$$

Result (type 5, 93 leaves):

$$\frac{1}{b^2 (a+bx)^{1/6}} (c+dx)^{1/6} \left( -6bc + 7ad + bdx + 7(bc - ad) \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/6} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad} \right] \right)$$

**Problem 1833: Result unnecessarily involves higher level functions.**

$$\int \frac{(c+dx)^{1/6}}{(a+bx)^{7/6}} dx$$

Optimal (type 3, 332 leaves, 13 steps):

$$\begin{aligned} & -\frac{6(c+dx)^{1/6}}{b(a+bx)^{1/6}} + \frac{\sqrt{3} d^{1/6} \text{ArcTan} \left[ \frac{1}{\sqrt{3}} - \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3} b^{1/6}(c+dx)^{1/6}} \right]}{b^{7/6}} - \\ & \frac{\sqrt{3} d^{1/6} \text{ArcTan} \left[ \frac{1}{\sqrt{3}} + \frac{2d^{1/6}(a+bx)^{1/6}}{\sqrt{3} b^{1/6}(c+dx)^{1/6}} \right]}{b^{7/6}} + \frac{2 d^{1/6} \text{ArcTanh} \left[ \frac{d^{1/6}(a+bx)^{1/6}}{b^{1/6}(c+dx)^{1/6}} \right]}{b^{7/6}} - \\ & \frac{d^{1/6} \text{Log} \left[ b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} - \frac{b^{1/6} d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}} \right]}{2 b^{7/6}} + \frac{d^{1/6} \text{Log} \left[ b^{1/3} + \frac{d^{1/3}(a+bx)^{1/3}}{(c+dx)^{1/3}} + \frac{b^{1/6} d^{1/6}(a+bx)^{1/6}}{(c+dx)^{1/6}} \right]}{2 b^{7/6}} \end{aligned}$$

Result (type 5, 74 leaves):

$$\frac{1}{b(a+bx)^{1/6}} (c+dx)^{1/6} \left( -1 + \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/6} \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{1}{6}, \frac{7}{6}, \frac{b(c+dx)}{bc-ad} \right] \right)$$

**Problem 1843: Result more than twice size of optimal antiderivative.**

$$\int \frac{1}{(a+bx)^{7/6} (c+dx)^{19/6}} dx$$

Optimal (type 5, 82 leaves, 2 steps):

$$-\frac{6b^2 \left( \frac{b(c+dx)}{bc-ad} \right)^{1/6} \text{Hypergeometric2F1} \left[ -\frac{1}{6}, \frac{19}{6}, \frac{5}{6}, -\frac{d(a+bx)}{bc-ad} \right]}{(bc-ad)^3 (a+bx)^{1/6} (c+dx)^{1/6}}$$

Result (type 5, 179 leaves):

$$\begin{aligned} & \left( -30(a^3 d^3 - a^2 b d^2 (5c + 2dx)) + \right. \\ & \quad \left. a b^2 d (23c^2 + 36cdx + 16d^2 x^2) + b^3 (13c^3 + 62c^2 dx + 80cd^2 x^2 + 32d^3 x^3) \right) + \\ & \quad 768 b^3 \left( \frac{d(a+bx)}{-bc+ad} \right)^{1/6} (c+dx)^3 \text{Hypergeometric2F1} \left[ \frac{1}{6}, \frac{5}{6}, \frac{11}{6}, \frac{b(c+dx)}{bc-ad} \right] \Big/ \\ & (65(bc-ad)^4 (a+bx)^{1/6} (c+dx)^{13/6}) \end{aligned}$$



**Problem 1850: Unable to integrate problem.**

$$\int \frac{(a + b x)^m}{(c + d x)^2} dx$$

Optimal (type 5, 52 leaves, 1 step):

$$\frac{b (a + b x)^{1+m} \text{Hypergeometric2F1}\left[2, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^2 (1 + m)}$$

Result (type 8, 17 leaves):

$$\int \frac{(a + b x)^m}{(c + d x)^2} dx$$

**Problem 1851: Unable to integrate problem.**

$$\int \frac{(a + b x)^m}{(c + d x)^3} dx$$

Optimal (type 5, 54 leaves, 1 step):

$$\frac{b^2 (a + b x)^{1+m} \text{Hypergeometric2F1}\left[3, 1 + m, 2 + m, -\frac{d (a + b x)}{b c - a d}\right]}{(b c - a d)^3 (1 + m)}$$

Result (type 8, 17 leaves):

$$\int \frac{(a + b x)^m}{(c + d x)^3} dx$$

**Problem 1857: Unable to integrate problem.**

$$\int \frac{(c + d x)^n}{(a + b x)^2} dx$$

Optimal (type 5, 51 leaves, 1 step):

$$\frac{d (c + d x)^{1+n} \text{Hypergeometric2F1}\left[2, 1 + n, 2 + n, \frac{b (c + d x)}{b c - a d}\right]}{(b c - a d)^2 (1 + n)}$$

Result (type 8, 17 leaves):

$$\int \frac{(c + d x)^n}{(a + b x)^2} dx$$

### Problem 1858: Unable to integrate problem.

$$\int \frac{(c+dx)^n}{(a+bx)^3} dx$$

Optimal (type 5, 54 leaves, 1 step):

$$\frac{d^2 (c+dx)^{1+n} \text{Hypergeometric2F1}\left[3, 1+n, 2+n, \frac{b(c+dx)}{bc-ad}\right]}{(bc-ad)^3 (1+n)}$$

Result (type 8, 17 leaves):

$$\int \frac{(c+dx)^n}{(a+bx)^3} dx$$

### Problem 1864: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a+bx)^{1+n} (c+dx)^{-n} dx$$

Optimal (type 5, 72 leaves, 2 steps):

$$\frac{1}{b(2+n)} (a+bx)^{2+n} (c+dx)^{-n} \left( \frac{b(c+dx)}{bc-ad} \right)^n \text{Hypergeometric2F1}\left[n, 2+n, 3+n, -\frac{d(a+bx)}{bc-ad}\right]$$

Result (type 6, 200 leaves):

$$a (a+bx)^n (c+dx)^{-n} \left( \left( 3bcx^2 \text{AppellF1}\left[2, -n, n, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) / \left( 6ac \text{AppellF1}\left[2, -n, n, 3, -\frac{bx}{a}, -\frac{dx}{c}\right] + 2nx \right. \right. \\ \left. \left. \left( bc \text{AppellF1}\left[3, 1-n, n, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] - ad \text{AppellF1}\left[3, -n, 1+n, 4, -\frac{bx}{a}, -\frac{dx}{c}\right] \right) \right) - \right. \\ \left. \frac{\left( \frac{d(a+bx)}{-bc+ad} \right)^{-n} (c+dx) \text{Hypergeometric2F1}\left[1-n, -n, 2-n, \frac{b(c+dx)}{bc-ad}\right]}{d(-1+n)} \right)$$

### Problem 1865: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int (a+bx)^{2+n} (c+dx)^{-n} dx$$

Optimal (type 5, 72 leaves, 2 steps):

$$\frac{1}{b(3+n)} (a+bx)^{3+n} (c+dx)^{-n} \left( \frac{b(c+dx)}{bc-ad} \right)^n \text{Hypergeometric2F1}\left[n, 3+n, 4+n, -\frac{d(a+bx)}{bc-ad}\right]$$

Result (type 6, 317 leaves):

$$\begin{aligned}
 & a (a + b x)^n (c + d x)^{-n} \\
 & \left( \left( 3 a b c x^2 \operatorname{AppellF1}\left[2, -n, n, 3, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) / \left( 3 a c \operatorname{AppellF1}\left[2, -n, n, 3, -\frac{b x}{a}, -\frac{d x}{c}\right] + n x \right. \right. \\
 & \quad \left. \left. \left( b c \operatorname{AppellF1}\left[3, 1-n, n, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] - a d \operatorname{AppellF1}\left[3, -n, 1+n, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) \right) + \right. \\
 & \quad \left. \left( 4 b^2 c x^3 \operatorname{AppellF1}\left[3, -n, n, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) / \right. \\
 & \quad \left. \left( 12 a c \operatorname{AppellF1}\left[3, -n, n, 4, -\frac{b x}{a}, -\frac{d x}{c}\right] + 3 b c n x \operatorname{AppellF1}\left[4, 1-n, n, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] - \right. \right. \\
 & \quad \left. \left. 3 a d n x \operatorname{AppellF1}\left[4, -n, 1+n, 5, -\frac{b x}{a}, -\frac{d x}{c}\right] \right) - \frac{1}{d (-1+n)} \right. \\
 & \quad \left. a \left( \frac{d (a + b x)}{-b c + a d} \right)^{-n} (c + d x) \operatorname{Hypergeometric2F1}\left[1-n, -n, 2-n, \frac{b (c + d x)}{b c - a d}\right] \right)
 \end{aligned}$$

**Problem 1882: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^m (a c (1 + m) + b c (2 + m) x)^{-3-m} dx$$

Optimal (type 3, 95 leaves, 2 steps):

$$- \frac{(a + b x)^{1+m} (a c (1 + m) + b c (2 + m) x)^{-2-m}}{a b c (2 + m)} + \frac{(a + b x)^{1+m} (a c (1 + m) + b c (2 + m) x)^{-1-m}}{a^2 b c^2 (1 + m) (2 + m)}$$

Result (type 5, 82 leaves):

$$- \frac{1}{a^3 b c^3 (1 + m)} (a + b x)^{1+m} (c (a (1 + m) + b (2 + m) x))^{-m} \\
 \left( -1 - m - \frac{b (2 + m) x}{a} \right)^m \operatorname{Hypergeometric2F1}\left[1 + m, 3 + m, 2 + m, \frac{(2 + m) (a + b x)}{a}\right]$$

**Problem 1884: Result unnecessarily involves higher level functions.**

$$\int (a + b x)^{\frac{-2 b c + a d}{b c - a d}} (c + d x)^{\frac{b c - 2 a d}{-b c + a d}} dx$$

Optimal (type 3, 97 leaves, 2 steps):

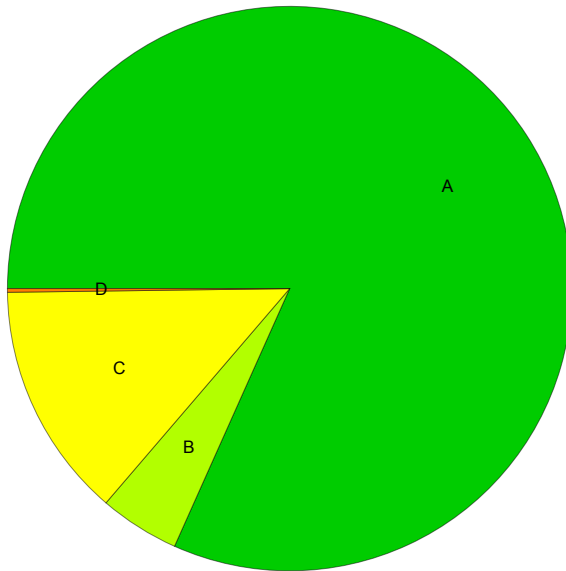
$$- \frac{(a + b x)^{\frac{b c}{b c - a d}} (c + d x)^{\frac{a d}{b c - a d}}}{b c} + \frac{(a + b x)^{\frac{a d}{b c - a d}} (c + d x)^{\frac{a d}{b c - a d}}}{a b c}$$

Result (type 5, 159 leaves):

$$\frac{1}{a d^2} (b c - a d) (a + b x)^{\frac{-2 b c + a d}{b c - a d}} \left( \frac{d (a + b x)}{-b c + a d} \right)^{\frac{-2 b c + a d}{-b c + a d}} (c + d x)^{\frac{a d}{b c - a d}} \\
 \operatorname{Hypergeometric2F1}\left[\frac{a d}{b c - a d}, \frac{-2 b c + a d}{-b c + a d}, \frac{b c}{b c - a d}, \frac{b (c + d x)}{b c - a d}\right]$$

## Summary of Integration Test Results

1917 integration problems



- A - 1566 optimal antiderivatives
- B - 88 more than twice size of optimal antiderivatives
- C - 259 unnecessarily complex antiderivatives
- D - 4 unable to integrate problems
- E - 0 integration timeouts